Ontario

A Guide to Effective Instruction in Mathematics

Kindergarten to Grade 3

Patterning and Algebra

2007

Ontario Education excellence for all

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Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

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Introduction

This document is a practical guide that teachers will find useful in helping students achieve the curriculum expectations for mathematics outlined in *The Kindergarten Program, 2006* (on page 47, under the subheading "Patterning") and the expectations outlined in the Patterning and Algebra strand for Grades 1 to 3 in *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. It is a companion document to *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The expectations outlined in the curriculum documents describe the knowledge and skills that students are expected to acquire by the end of each grade. In *Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario* (Expert Panel on Early Math in Ontario, 2003), effective instruction is identified as critical to the successful learning of mathematical knowledge and skills, and the components of an effective program are described. As part of the process of implementing the panel's vision of effective mathematics instruction for Ontario, *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006* provides a framework for teaching mathematics. This framework includes specific strategies for developing an effective program and for creating a community of learners in which students' mathematical thinking is nurtured. The strategies described in the guide focus on the "big ideas" inherent in the expectations; on problem solving as the main context for mathematical activity; and on communication, especially student talk, as the conduit for sharing and developing mathematical thinking. The guide also provides strategies for assessment, the use of manipulatives, and home connections.

Purpose and Features of the Document

The present document was developed to provide practical applications of the principles and theories behind good instruction that are elaborated in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The present document provides:

- an overview of each of the big ideas in the Patterning and Algebra strand;
- four appendices (Appendices A–D), one for each grade from Kindergarten to Grade 3, which provide learning activities that introduce, develop, or help to consolidate some aspect of each big idea. These learning activities reflect the instructional practices recommended in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*;
- an appendix (Appendix E) that lists the curriculum expectations in the Patterning and Algebra strand under the big idea to which they correspond. This clustering of expectations around each of the two big ideas allows teachers to concentrate their programming on the big ideas of the strand while remaining confident that the full range of curriculum expectations is being addressed;
- a glossary that provides definitions of mathematical terms used in this document.

"Big Ideas" in the Curriculum for Kindergarten to Grade 3

In developing a mathematics program, it is vital to concentrate on important mathematical concepts, or "big ideas", and the knowledge and skills that go with those concepts. Programs that are organized around big ideas and focus on problem solving provide cohesive learning opportunities that allow students to explore concepts in depth.

> All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the "big ideas", or key principles, of mathematics, such as pattern or relationship. (Ontario Ministry of Education, 2005, p. 25)

Students are better able to see the connections in mathematics and thus to *learn* mathematics when it is organized in big, coherent "chunks". In organizing a mathematics program, teachers should concentrate on the big ideas in mathematics and view the expectations in the curriculum policy documents for Kindergarten and Grades 1 to 3 as being clustered around those big ideas.

The clustering of expectations around big ideas provides a focus for student learning and for teacher professional development in mathematics. Teachers will find that investigating and discussing effective teaching strategies for a big idea is much more valuable than trying to determine specific strategies and approaches to help students achieve individual expectations. In fact, using big ideas as a focus helps teachers see that the concepts represented in the curriculum expectations should not be taught as isolated bits of information but rather as a network of interrelated concepts.

In building a program, teachers need a sound understanding of the key mathematical concepts for their students' grade level and a grasp of how those concepts connect with students' prior and future learning (Ma, 1999). They need to understand the "conceptual structure and basic attitudes of mathematics inherent in the elementary curriculum" (p. xxiv) and to know how best to teach the concepts to students. Concentrating on developing this knowledge and understanding will enhance effective teaching.

Focusing on the big ideas provides teachers with a global view of the concepts represented in the strand. The big ideas also act as a "lens" for:

- making instructional decisions (e.g., choosing an emphasis for a lesson or set of lessons);
- identifying prior learning;
- looking at students' thinking and understanding in relation to the mathematical concepts addressed in the curriculum (e.g., making note of the ways in which a student creates patterns, using concrete materials);
- collecting observations and making anecdotal records;
- providing feedback to students;
- determining next steps;
- communicating concepts and providing feedback on students' achievement to parents[1] (e.g., in report card comments).

Teachers are encouraged to focus their instruction on the big ideas of mathematics. By clustering expectations around a few big ideas, teachers can teach for depth of understanding. This document provides models for clustering the expectations around a few major concepts and includes activities that foster understanding of the big ideas in Patterning and Algebra. Teachers can use these models in developing other lessons in Patterning and Algebra, as well as lessons in other strands of mathematics.

1. In this document, *parent(s)* refers to parent(s) and guardian(s).

The "Big Ideas" in Patterning and Algebra

One of the central themes in mathematics is the study of patterns and relationships. This study requires students to recognize, describe, and generalize patterns and to build mathematical models to simulate the behaviour of real-world phenomena that exhibit observed patterns.

(Ontario Ministry of Education, 2005, p. 9)

Overview

The Patterning and Algebra strand of the Ontario mathematics curriculum involves the study of patterns in shapes, designs, movement, and numbers. In the primary grades, students identify, create, and describe a variety of patterns, and make generalizations about relationships within and between patterns. Young students also learn that patterns and numerical relationships can be represented symbolically, and begin to represent mathematical ideas, using simple numerical expressions and equations.

This section focuses on the two big ideas that form the basis of the curriculum expectations in Patterning and Algebra for Kindergarten to Grade 3. An understanding of these big ideas assists teachers in providing instructional and assessment opportunities that promote student learning of important concepts in Patterning and Algebra.

The big ideas or major concepts in Patterning and Algebra are the following:

- patterns and relationships
- expressions and equality

Teachers should recognize that these big ideas are conceptually related and interdependent, and that many instructional experiences reflect both big ideas. For example, as students explore mathematical patterns and relationships, they develop an understanding of how expressions and equations can be used to generalize, represent, and explain such patterns.

The discussion of each big idea in this section contains:

- an **overview**, which includes a general discussion of the development of the big idea in the primary grades, an explanation of some of the key concepts inherent in the big idea, and in some instances additional background information on the concepts for the teacher;
- **grade-specific descriptions** of (1) characteristics of learning evident in students who have been introduced to the concepts addressed in the big idea, and (2) instructional strategies that will support those learning characteristics. In order to address a range of student learning needs, teachers should examine instructional strategies for grade levels other than their own.

General Principles of Instruction

The following principles of instruction are relevant in teaching Patterning and Algebra in the primary grades:

- **Student talk is important.** Students need to talk about and talk through mathematical concepts, with one another and with the teacher.
- **Representations of concepts promote understanding and communication.** In Patterning and Algebra, concepts can be represented in various ways (e.g., through the use of manipulatives, pictures, words, symbols). Teachers need to help students make connections between different representations of a mathematical concept (e.g., show them how to use concrete materials, a diagram, and a number sequence to represent a growing pattern).
- **Students learn through problem solving.** Problem-solving situations provide students with a context and a meaningful purpose for reasoning about mathematical concepts and ideas. As well, organizing learning activities within a three-part lesson based on problem solving prompts students to engage in a problem-solving process of learning mathematics. The main parts of the three-part lesson structure recommended in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006* are Getting Started, Working on It, and Reflecting and Connecting. For examples of the three-part lesson structure, see the learning activities in this guide.
- **Students need frequent experiences using a variety of learning strategies (e.g., investigations, problem-solving activities, games) and resources (e.g., pattern blocks, interlocking cubes, hundreds charts, calculators).** A variety of learning strategies should be used in instruction to address the learning styles of all students.

- **Teachers can help students acquire mathematical language by using correct mathematical vocabulary themselves.** Beginning in Kindergarten, teachers should model appropriate mathematical terminology and encourage students to use mathematical vocabulary that will allow them to express themselves clearly and precisely.

Working Towards Equitable Outcomes for Diverse Students

All students, whatever their socio-economic, ethnocultural, or linguistic background, must have opportunities to learn and to grow, both cognitively and socially. When students can make personal connections to their learning, and when they feel secure in their learning environment, their true capacity will be realized in their achievement. A commitment to equity and inclusive instruction in Ontario classrooms is therefore critical to enabling all students to succeed in school and, consequently, to become productive and contributing members of society.

To create effective conditions for learning, teachers must take care to avoid all forms of bias and stereotyping in resources and learning activities, which can quickly alienate students and limit their learning. Teachers should be aware of the need to provide a variety of experiences and to encourage multiple perspectives, so that the diversity of the class is recognized and all students feel respected and valued. Learning activities and resources for teaching mathematics should be inclusive, providing examples and illustrations and using approaches that recognize the range of experiences of students with diverse backgrounds, knowledge, skills, interests, and learning styles.

The following are some strategies for creating a learning environment that acknowledges and values the diversity of students, and enables them to participate fully in the learning experience:

- providing mathematics problems with situations and contexts that are meaningful to all students (e.g., problems that reflect students' interests, home-life experiences, and cultural backgrounds and that stimulate their curiosity and spirit of enquiry);
- using mathematics examples drawn from diverse cultures, including those of Aboriginal peoples;
- using children's literature that reflects various cultures and customs as a source of mathematics examples and situations;

- understanding and acknowledging customs and adjusting teaching strategies, as necessary. For example, a student may come from a culture in which it is considered inappropriate for a child to ask for help, express opinions openly, or make direct eye contact with an adult;
- considering the appropriateness of references to holidays, celebrations, and traditions;
- providing clarification if the context of a learning activity is unfamiliar to students (e.g., describing or showing a food item that may be new to some students);
- evaluating the content of mathematics textbooks, children's literature, and supplementary materials for cultural or gender bias;
- designing learning and assessment activities that allow students with various learning styles (e.g., auditory, visual, tactile/kinaesthetic) to participate meaningfully;
- providing opportunities for students to work both independently and interdependently with others;
- providing opportunities for students to communicate orally and in writing in their home language (e.g., pairing English language learners with a first-language peer who also speaks English);
- using diagrams, pictures, manipulatives, sounds, and gestures to clarify mathematical vocabulary that may be new to English language learners.

For a full discussion of equity and diversity in the classroom, as well as a detailed checklist for providing inclusive mathematics instruction, see pages 34 – 40 in Volume 1 of *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006.*

Patterns and Relationships

Mathematics is the science and language of patterns. Thinking about patterns helps children make sense of mathematics. They learn that mathematics is not a set of unrelated facts and procedures; instead, recognizing and working with patterns helps young children predict what will happen, talk about relationships, and see connections between mathematics concepts and their world.

(Copley, 2000, p. 83)

Overview

Young children are naturally interested in patterns. They enjoy patterns in rhymes, chants, songs, poems, and stories, and they experiment with creating patterns, using play materials. They examine patterns in wallpaper, fabric, and geometric designs, and recognize patterns in their lives (e.g., the sequence of daily routines) and in nature (e.g., the cycle of the seasons). When students enter school, they continue to explore patterns in mathematics and in other subject areas.

This segment is unavailable due to copyright restrictions. To see the uncut text, refer to the printed version.

Developing an understanding of patterns and relationships is central to learning mathematics. In the primary grades, students identify, extend, and create a variety of patterns, using objects, actions, sounds, pictures, letters, and numbers. Through experiences in exploring and discussing patterns, young students begin to make conjectures and generalizations about mathematical relationships. Learning opportunities in later grades help students analyse increasingly complex mathematical relationships and represent these relationships algebraically.

The following are key points that can be made about patterns and relationships in the primary grades:

- Experience with a wide variety of patterns helps students recognize relationships within and between patterns.

- Growing and shrinking patterns involve an increase or a decrease in elements as the pattern continues. The increase or decrease in elements can be described numerically.
- An understanding of patterns in numbers and operations contributes to the development of algebraic thinking.

Experiences With a Variety of Patterns

Opportunities to explore various patterns allow students to recognize the repetitive nature of patterns. These experiences also help students make predictions and conjectures about what comes next in a pattern without actually having to create the next part of the pattern. Through classroom experiences in identifying, extending, and creating repeating patterns, students develop the ability to describe the patterns, using informal language, and to recognize relationships within and between patterns.

Students develop an awareness and understanding of patterns and pattern relationships through experiences with patterns in a variety of forms.

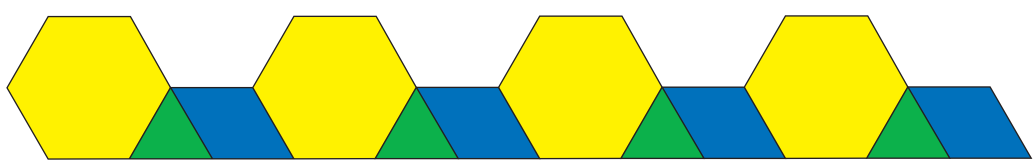
People patterns: boy – girl – boy – girl – boy – girl

Patterns involving actions: jump – jump – hop – jump – jump – hop – jump – jump – hop

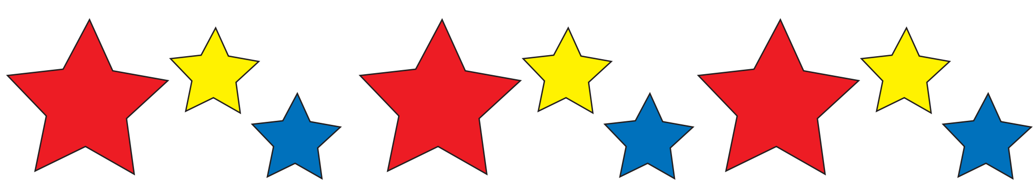
Patterns with sounds: snap – clap – clap – snap – clap – clap – snap – clap – clap

Patterns with objects:

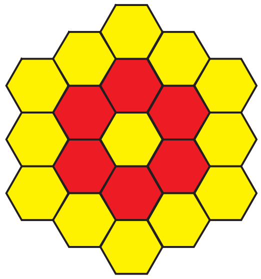
Patterns with geometric shapes:



Patterns with pictures or symbols:



Patterns in designs:

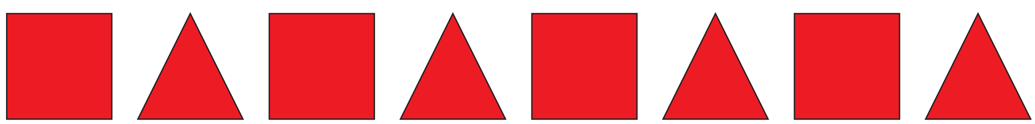


Number patterns: 11, 22, 33, 44, 55, . . .

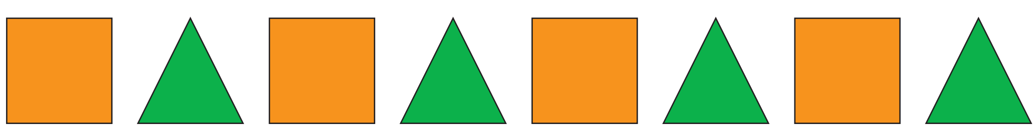
Patterns with letters: ABCCC, ABCCC, ABCCC

Throughout the primary grades, students deepen their understanding of patterns and relationships as they investigate repeating patterns, using a variety of concrete materials. Simple repeating patterns involving changes in colour (e.g., red cube, blue cube, red cube, blue cube, red cube, blue cube) are easiest for students to identify and create. As a result of experiences in using various materials to explore a variety of patterns, students learn to identify, describe, and extend increasingly complex repeating patterns. The complexity of a repeating pattern depends on the following factors:

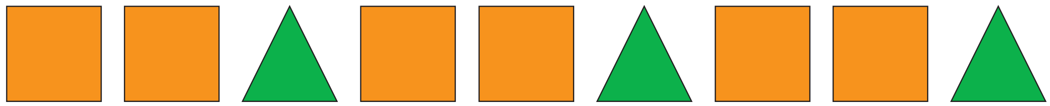
- **The nature of the attribute(s) in the pattern.** Certain attributes of patterns are clear and noticeable (e.g., colour). A pattern is said to increase in complexity if it involves changes to one or more attributes that are less obvious (e.g., size, shape, texture, thickness, orientation, or material).
- **The number of changing attributes in a pattern.** Simple patterns involve changes to one attribute. For example, in the repeating pattern illustrated below, shape is the changing attribute; size and colour remain constant.



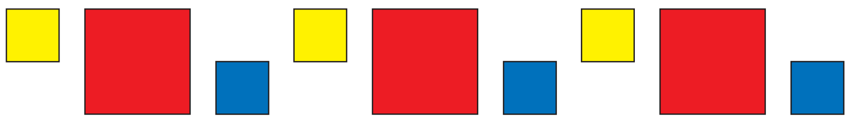
More complex patterns involve changes to two or more attributes. In the following example, the pattern involves changes to both shape (square, triangle) and colour (orange, green).



- **The number of elements in the core of the pattern.** The *core* (also called *stem*) refers to the part of the pattern that continuously recurs. It is composed of *elements*, that is, the specific items within a pattern. The patterns in the two preceding illustrations involve a core with two elements (square, triangle). The following pattern is more complex: it involves a core with three elements (i.e., square, square, triangle).



- **The number of changing attributes within the core.** The following pattern is more complex than those illustrated above because of the changes to the attributes (i.e., colour, size, orientation) within the pattern core.



A variety of learning experiences help students develop concepts about patterns, recognize relationships within and between patterns, and make generalizations about patterns (e.g., show how a pattern can be extended). It is important that students have many opportunities to investigate patterns, using a variety of concrete materials. Unlike paper-and-pencil tasks in which students colour shapes or draw missing elements in a pattern, experiences with concrete materials allow students to manipulate and change elements within a pattern easily and without the fear of making mistakes.

Students learn about patterns and pattern relationships through the following experiences:

Identifying patterns. Patterns are everywhere, and teachers should encourage students to identify patterns that occur in their environment, in nature, in literature, in classroom routines, and so on. Having young students explain patterns in their own language (e.g., "It's the same two colours each time" or "This part keeps repeating over and over") helps them understand the repetitive nature of patterns.

Reading patterns. For young students, reading a pattern involves pointing at and naming each element (e.g., "orange square, orange square, green triangle, orange square, orange square, green triangle, . . . "). Reading a pattern helps students recognize the changing attributes within the pattern, as well as the number and kinds of elements in the pattern core.

Describing patterns. After reading a pattern, students are better able to describe it. Describing patterns involves identifying the changing attributes, the number of elements in the pattern core, and the kinds of elements in the core. Teachers should encourage students to use appropriate mathematical language (e.g., *core*, *position*, *repeating*) to describe patterns.

Extending patterns. Students learn that patterns can be extended because they are repetitive by nature. Having students extend patterns (e.g., continue a given pattern, using pattern blocks) helps them focus on the core of the pattern and the way in which the core repeats as the pattern continues.

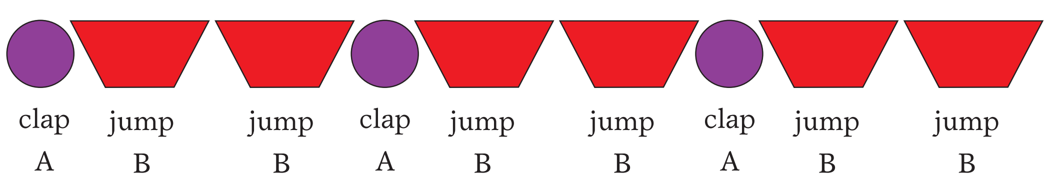
Determining pattern rules. Experiences in describing and extending patterns help students make generalizations about the composition of different patterns. Students can express these generalizations as pattern rules, that is, concise descriptions of how a pattern repeats. In the primary grades, students express pattern rules informally (e.g., "In this pattern, there are two green triangles and an orange square, and that part of the pattern keeps repeating").

Translating patterns. Given many opportunities to identify, read, describe, and extend patterns, students learn that they can represent the same pattern structure in a variety of ways. For example, students might observe that the following patterns involve the same structure:

- step – hop – hop – step – hop – hop – step – hop – hop
- triangle – square – square – triangle – square – square – triangle – square – square
- up – down – down – up – down – down – up – down – down

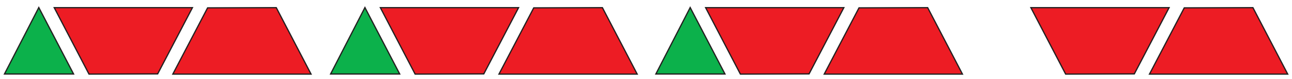
Recognizing that the same pattern can be represented in different ways is a significant step in students' mathematical thinking (Van de Walle & Folk, 2005). Students learn that pattern structures can be generalized and represented by a series of letters. For example, the preceding patterns can be represented as ABB, ABB, ABB.

Students should be given opportunities to translate patterns from one form to another. A translation exercise might involve using two or more different kinds of concrete materials to represent the same pattern structure (e.g., translating a pattern involving interlocking cubes into a pattern involving colour tiles). Students might also translate a pattern from one mode to another. In the following example, shapes, actions, and letters are used to represent the same pattern.



Teachers can emphasize the similarities and differences between translated patterns by asking: "How are these patterns the same? How are they different?" Comparing translated patterns highlights the equivalence of their underlying mathematical structure, even though the modes of representation differ (e.g., circle – trapezoid – trapezoid versus clap – jump – jump).

Finding missing elements in patterns. A worthwhile task for students is to identify a missing element in a pattern. For example, students might determine the missing element in the following pattern:

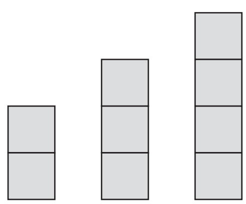


Having students determine a missing element in a pattern extends their understanding of the relationships between the elements in a pattern and helps them analyse pattern structures.

Note: In classroom activities, teachers should present at least three complete repetitions of a pattern core to help students recognize the structure of the pattern.

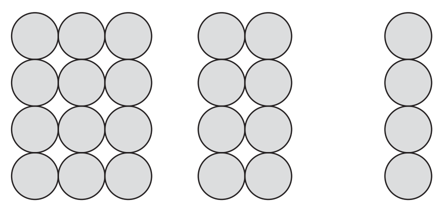
Growing and Shrinking Patterns

In growing and shrinking patterns, the number of elements increases or decreases from one term to the next.



Term 1 **Term 2** **Term 3**

Growing Pattern

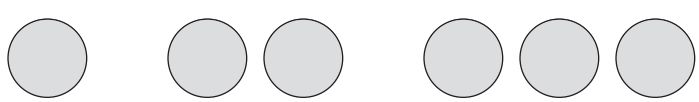


Term 1 **Term 2** **Term 3**

Shrinking Pattern

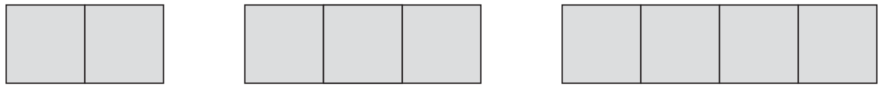
Growing and shrinking patterns are not a focus in learning activities until Grade 2. Students explore growing and shrinking patterns, using a variety of materials (e.g., pattern blocks, colour tiles, interlocking cubes, counters). As students examine, create, and extend simple growing and shrinking patterns, they observe that there is a consistent increase or decrease of elements from term to term.

A simple growing pattern begins with one element in the first term and increases by one element in each subsequent term.



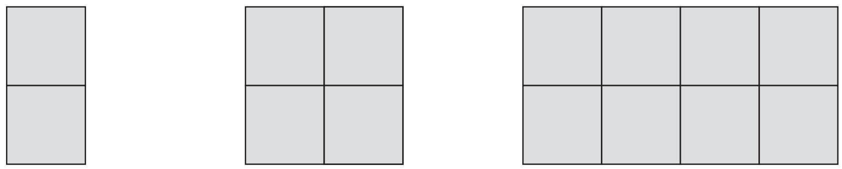
Term 1 **Term 2** **Term 3**

More complex patterns have more than one element in the first term, with one or more elements added at each new term. In the following example, the first term begins with two squares, and one square is added at each subsequent term.



Term 1 **Term 2** **Term 3**

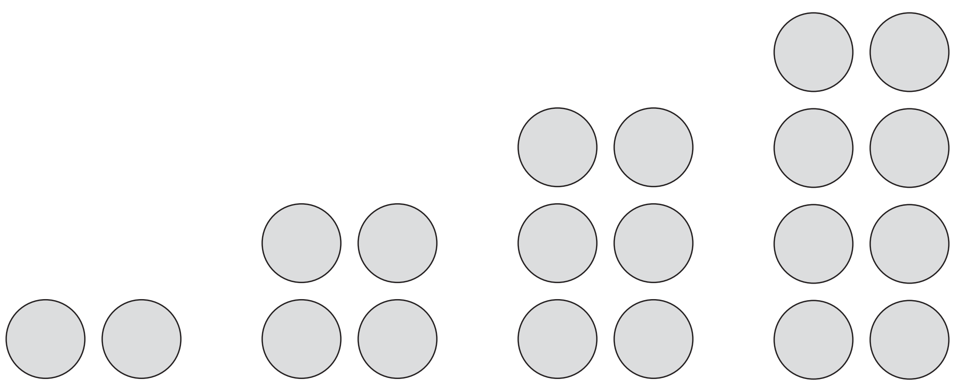
Growing patterns can also involve multiplicative relationships. In the following example, the number of elements is doubled at each term.



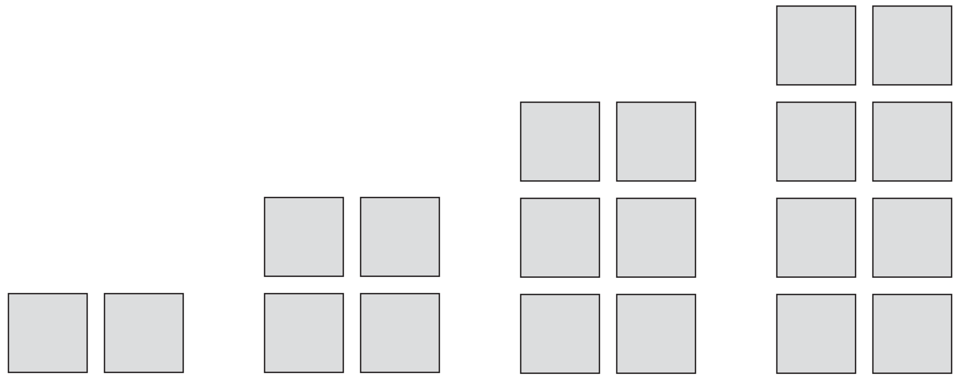
Term 1 **Term 2** **Term 3**

The complexity of shrinking patterns depends on the number of elements that are removed at each progressive term, and on the operation (e.g., subtraction, division) performed from term to term.

In Grades 2 and 3, students represent growing or shrinking patterns in a variety of ways. Translating a pattern from one form to another helps students recognize the consistent change that occurs from term to term. For example, students might create the following pattern, using counters, and then represent it in different ways:



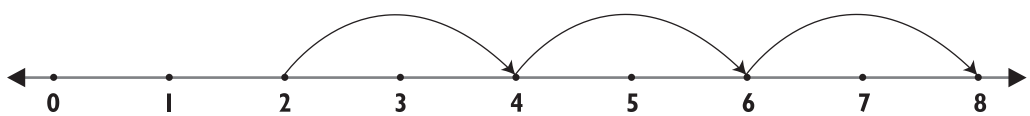
Using a different concrete material. Using a different concrete material (e.g., colour tiles) to recreate a pattern helps students recognize how the pattern grows (or shrinks) and recognize the physical arrangement of the pattern.



Using a different mode (e.g., actions, colours, sounds, letters). Translating a pattern into another mode focuses students' attention on the number of elements in each new term.

jump – jump
jump – jump – jump – jump
jump – jump – jump – jump – jump – jump
jump – jump – jump – jump – jump – jump – jump – jump

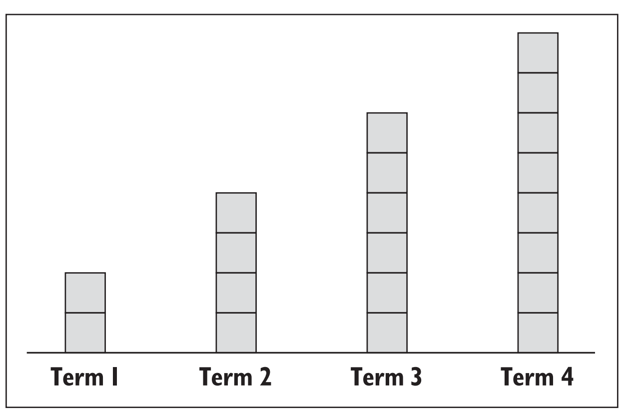
Using a number line. A number line illustrates the rate of increase (or decrease) in a pattern.



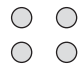
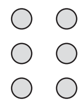
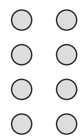
Using a number sequence. Students can relate patterns that involve addition and subtraction to skip-counting sequences.

2, 4, 6, 8

Using a bar graph. By constructing simple bar graphs that represent growing or shrinking patterns, students can observe the increase or decrease in the number of elements from term to term.



Using a table. Information about the pattern can be recorded in a table. The arithmetic relationship between terms (e.g., + 2) can be indicated at the side of the table.

Term	Diagram	Number of Counters
1		2
2		4
3		6
4		8

+ 2
+ 2
+ 2

The description of the way in which a pattern changes from term to term is known as a *recursive relationship*. When students in the primary grades recognize how a growing or shrinking pattern evolves, they describe the recursive relationship, using informal language (e.g., "You add two more counters every time"). By having students represent patterns in tables, as illustrated above, teachers can help students recognize that the rule for a growing or shrinking pattern can be expressed arithmetically (e.g., + 2).

Term	Number of Counters
1	2
2	4
3	6
4	8

× 2

Representing patterns in tables also prepares students for analysing increasingly complex pattern relationships in later grades. Tables help students identify functional relationships – for example, the relationship between the term number and the number of elements at that term. In the table to the left, the number of counters at any term is double the term number. This functional relationship between term number and number of counters can be expressed as "multiply by 2", and can be generalized: If the term number is represented by n, then the number of counters is $2 \times n$, or $2n$. Students begin to develop this kind of algebraic thinking in the junior grades.

Number Patterns

As students develop an understanding of number, they recognize that patterns exist in our number system and in number operations. Recognizing these patterns allows students to understand that numbers and operations work in consistent and predictable ways, and helps them make generalizations about numerical relationships. Developing a sense about numbers and operations, and the patterns inherent in them, provides a foundation for students' development of algebraic thinking.

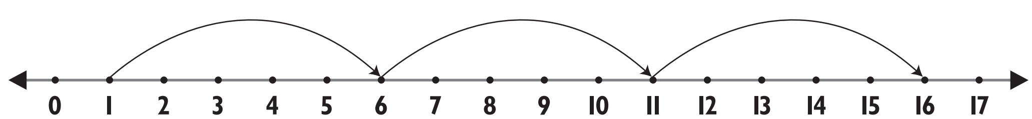
Students require many opportunities to explore patterns in numbers and in number operations. Teachers should highlight number and operation patterns whenever they arise. The following are among the many number patterns that students explore in the primary grades:

Counting patterns. Students learn that there is a repetitive pattern of the count 1 to 9 that extends through the number system. Knowing this pattern allows students to count forwards and backwards.

Skip-counting patterns. Students recognize patterns in skip counting. For example, they see that counting by 2's from 2 involves even numbers, and that counting by 2's from 1 involves odd numbers. They also learn to recognize other skip-counting patterns (e.g., counting by 5's from 5 involves numbers that have 0 or 5 in the ones place; counting by 10's from 10 involves numbers that have 0 in the ones place).

Experiences in skip counting help students extend growing and shrinking number patterns (e.g., determine the next three terms in the pattern 3, 6, 9, 12, . . .), and create number patterns, given an oral or a written description of a number pattern (e.g., create a number pattern that starts at 1 and grows by adding 5 each time).

Skip-counting sequences can be represented on a number line.



Patterns in a hundreds chart. Work with a hundreds chart allows students to recognize many numeric patterns. For example, a hundreds chart reveals the repetitive pattern of the digits of 1 to 9 in each decade, and of these digits in the tens place (e.g., 10, 20, 30, 40, . . .). Representing growing and shrinking patterns also results in visual patterns in a hundreds chart. The hundreds chart on the next page shows the patterns that result from beginning at 2 and repeatedly adding 2.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Base ten patterns. Experiences with base ten materials help students recognize patterns in our number system (e.g., 1 tens rod represents 10, 2 tens rods represent 20, 3 tens rods represent 30, and so on).

Place-value patterns. Work with place-value patterns allows students to represent numbers in different ways. For example, they can represent a two-digit number as different numbers of tens and ones.

37 = 3 tens + 7 ones
37 = 2 tens + 17 ones
37 = 1 ten + 27 ones
37 = 37 ones

Recognizing place-value patterns helps students understand how numbers can be regrouped in computations.

Patterns in addition and subtraction. Students learn to recognize patterns in basic addition and subtraction facts, for example, patterns in facts for 7:

0 + 7 = 7	7 – 7 = 0
1 + 6 = 7	7 – 6 = 1
2 + 5 = 7	7 – 5 = 2
3 + 4 = 7	7 – 4 = 3
4 + 3 = 7	7 – 3 = 4
5 + 2 = 7	7 – 2 = 5
6 + 1 = 7	7 – 1 = 6
7 + 0 = 7	7 – 0 = 7

Knowing patterns in basic facts allows students to add and subtract larger numbers.

2 + 7 = 9	9 – 7 = 2
12 + 7 = 19	19 – 7 = 12
22 + 7 = 29	29 – 7 = 22
32 + 7 = 39	39 – 7 = 32
42 + 7 = 49	49 – 7 = 42
52 + 7 = 59	59 – 7 = 52
62 + 7 = 69	69 – 7 = 62
72 + 7 = 79	79 – 7 = 72

Patterns in multiplication and division. As students learn basic multiplication and division facts, they observe patterns (e.g., the products of 5 have either 0 or 5 in the ones place).

1 x 5 = 5	5 ÷ 5 = 1
2 x 5 = 10	10 ÷ 5 = 2
3 x 5 = 15	15 ÷ 5 = 3
4 x 5 = 20	20 ÷ 5 = 4
5 x 5 = 25	25 ÷ 5 = 5
6 x 5 = 30	30 ÷ 5 = 6
7 x 5 = 35	35 ÷ 5 = 7

Knowing patterns in multiplication and division facts allows students to make generalizations about the divisibility of numbers in later grades (e.g., 385 is divisible by 5 because there is a 5 in the ones place).

Characteristics of Student Learning and Instructional Strategies by Grade

KINDERGARTEN

Characteristics of Student Learning

In general, students in Kindergarten:

- recognize, identify, and describe patterns in their environment (e.g., geometric patterns in fabrics and wallpaper);
- describe patterns in stories, songs, poems, and chants;
- recognize, identify, describe, and copy repeating patterns involving one attribute (e.g., colour, size, shape);
- develop an understanding of the repetitive nature of patterns (e.g., asked to describe why a design is a pattern, students may respond, "Because it's the same over and over again");

- extend repeating patterns involving one attribute, using a variety of concrete materials (e.g., colour tiles, interlocking cubes, attribute blocks);
- create repeating patterns (e.g., using concrete materials, using actions, using sounds);
- develop an understanding of counting patterns, and use this knowledge to learn the names of numbers in the decades (e.g., "twenty-one, twenty-two, twenty-three, . . ."; "thirty-one, thirty-two, thirty-three, . . .");
- begin to see patterns in a hundreds chart and on a number line.

Instructional Strategies

Students in Kindergarten benefit from the following instructional strategies:

- providing experiences, on an ongoing basis, with repeating patterns in a variety of forms. Such experiences include reading pattern stories, reciting chants, creating physical patterns (e.g., have students create a circle involving a sit-stand-sit-stand-sit-stand pattern), using concrete materials to build patterns, and discussing geometric patterns in fabrics and in art;
- having them read patterns (e.g., show a pattern made with colour tiles and have students point to and identify each element);
- having them look at various patterns and describe what is the same in them and what is different;
- discussing patterns and relationships within patterns (e.g., discuss the part of the pattern that repeats; identify the elements within the repeating part);
- providing many opportunities to explore and analyse the same patterns represented in different ways (e.g., examine a red-blue-red-blue-red-blue pattern, using both interlocking cubes and attribute blocks);
- describing patterns on the calendar (e.g., days of the week, months of the year);
- encouraging them to explain their patterning rules (e.g., "My rule is three red beads and one blue bead, and that keeps repeating over and over");
- exploring repeating patterns in the number system (e.g., how the ones digits repeat in each decade).

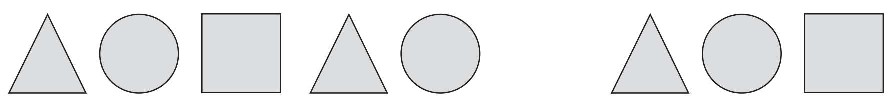
Grade 1

Characteristics of Student Learning

In general, students in Grade 1:

- recognize, identify, and describe patterns in their environment (e.g., geometric patterns in fabrics and wallpaper, numeric patterns on a calendar);
- describe patterns in stories, songs, poems, and chants;

- identify, describe, and extend repeating patterns involving one attribute (e.g., colour, size, shape, thickness, orientation);
- identify similarities in, or differences between, patterns (e.g., "This pattern has a blue triangle, a red triangle, and a blue triangle; but in this other pattern, there is a blue square, a red square, and a blue square");
- identify the core of patterns involving one attribute, and explain how the core repeats in a pattern (e.g., "In this pattern, circle-square-square repeats over and over again");
- begin to discuss pattern rules informally, and use pattern rules to extend patterns;
- begin to recognize that the same repeating pattern can exist in different forms. For example, a geometric pattern involving the repetition of blue triangle–green triangle and an action pattern involving the repetition of snap-clap are both AB patterns;
- extend repeating patterns involving one attribute, using a variety of concrete materials (e.g., colour tiles, interlocking cubes, attribute blocks);
- create repeating patterns (e.g., using concrete materials, using actions, using sounds);
- identify a missing element in a repeating pattern;



- recognize patterns and relationships in a hundreds chart (e.g., when counting is done by 5's, the numbers that are counted appear in two columns of the hundreds chart) and on number lines.

Instructional Strategies

Students in Grade 1 benefit from the following instructional strategies:

- providing multiple experiences, on an ongoing basis, with repeating patterns in a variety of forms. Such experiences include reading pattern stories, reciting chants, creating physical patterns (e.g., have students create a circle involving a sit-stand-sit-stand-sit-stand pattern), using concrete materials to build patterns, and discussing geometric patterns in fabrics and in art;
- providing many opportunities to create the same pattern, using different materials, and having students compare the patterns. For example, students might create a pattern, using attribute blocks (e.g., thick triangle, thin triangle, thick triangle, thin triangle, thick triangle, thin triangle); create a pattern that has the same structure as the pattern using attribute blocks (e.g., large circle,

small circle, large circle, small circle, large circle, small circle); and then compare the two patterns;

- discussing patterns and pattern relationships (e.g., discuss the core of a pattern; identify the elements within the core);
- providing many opportunities to explore simple patterns before investigating more complex patterns (e.g., have students describe and extend simple AB patterns before they explore patterns with structures such as ABB or AABB);
- exploring number patterns and relationships in a hundreds chart (e.g., count by 2′s 5′s, 10′s in a hundreds chart and discuss skip-counting patterns);
- encouraging them to explain their patterning rules (e.g., "My rule is 2 red cubes, 1 blue cube, and 1 green cube, and that part of the pattern keeps repeating");
- having them create patterns, given oral or written descriptions of the pattern (e.g., "Create an ABB pattern, using pattern blocks").

Grade 2

Characteristics of Student Learning

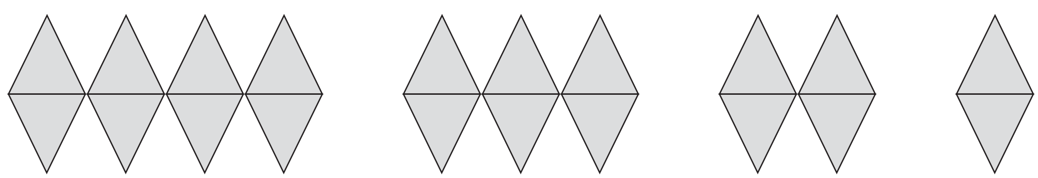
In general, students in Grade 2:

- recognize, identify, and describe patterns in their environment (e.g., geometric patterns in fabrics and in art, numeric patterns on a calendar);
- describe patterns in stories, songs, poems, and chants;
- identify, describe, extend, and create repeating patterns involving two attributes (e.g., colour *and* size);
- identify the core of patterns, and explain how the core repeats in a pattern (e.g., "In this pattern, circle-square-square repeats over and over again");
- identify missing elements in a pattern;
- explain pattern rules informally, and use pattern rules to extend patterns;
- recognize that the same repeating patterns can exist in many different forms (e.g., a repeating pattern involving pattern blocks can be translated into a pattern involving actions);
- begin to develop an understanding of growing and shrinking patterns, and begin to describe pattern rules informally (e.g., describe how a pattern changes as it progresses from one term to the next);
- describe number patterns and relationships in a hundreds chart, in addition charts, and on number lines.

Instructional Strategies

Students in Grade 2 benefit from the following instructional strategies:

- providing multiple experiences, on an ongoing basis, with repeating patterns in a variety of forms. Such experiences include reading pattern stories, reciting chants, using concrete materials to build patterns, discussing geometric patterns in fabrics and in art, and examining numeric patterns on a calendar (e.g., determine the pattern of dates for every Wednesday in a given month);
- providing opportunities to create patterns involving two attributes (e.g., size *and* texture), using a variety of concrete materials (e.g., attribute blocks);
- discussing patterns and pattern relationships (e.g., discuss the core of the pattern; identify elements within the core);
- providing many opportunities to explore simple growing and shrinking patterns before investigating more complex patterns (e.g., have students describe and extend growing patterns that involve increases of one element at each new term before they explore growing patterns that involve increases of two or more elements at each new term);
- providing many opportunities to create the same repeating, growing, or shrinking pattern, using different materials, and to compare the various patterns created;
- encouraging students to explain their patterning rules (e.g., "My rule is that I start at 2 and then skip to every second number");
- providing opportunities to extend repeating, growing, and shrinking patterns and to justify how they extended the patterns;
- representing geometric growing and shrinking patterns as a number sequence (e.g., represent the following geometric pattern as 8, 6, 4, 2);



- providing opportunities to examine growing and shrinking patterns, and to explain the change that occurs between terms, using informal language (e.g., "In this pattern, 2 triangles are taken away each time");
- having them predict the number of elements in a later term of a growing or shrinking pattern (e.g., examine the number of cubes in the first four terms of a pattern and predict the number of cubes at the sixth term);
- exploring number patterns and relationships in a hundreds chart (e.g., find skip-counting patterns; find the multiples of a number), in addition charts, and on number lines.

Characteristics of Student Learning

In general, students in Grade 3:

- recognize, identify, and describe patterns in their environment (e.g., geometric patterns in fabrics and in art, numeric patterns on a calendar);
- describe patterns in stories, songs, poems, and chants;
- identify, describe, extend, and create repeating patterns involving two attributes (e.g., size *and* texture);
- identify missing elements in a pattern;
- describe growing and shrinking patterns, and explain the recursive relationship in patterns (i.e., how the pattern changes from term to term);
- describe relationships and patterns in a hundreds chart, in addition and multiplication charts, and on number lines.

Instructional Strategies

Students in Grade 3 benefit from the following instructional strategies:

- providing multiple experiences, on an ongoing basis, with repeating, growing, and shrinking patterns in a variety of forms. Such experiences include reading pattern stories, using concrete materials to build patterns, discussing geometric patterns in fabrics and in art, and examining numeric patterns on a calendar (e.g., examine number patterns in columns of a calendar);
- providing opportunities to create patterns involving two attributes (e.g., shape *and* colour), using a variety of concrete materials;
- providing opportunities to extend growing and shrinking patterns, and to justify how they extended the patterns;
- having them represent growing or shrinking patterns, using number sequences, charts, number lines, and bar graphs;
- discussing strategies for finding recursive relationships (e.g., examine how a growing or shrinking pattern changes from term to term);
- providing opportunities to examine growing and shrinking patterns, and to explain the change that occurs between terms, using informal language (e.g., "In this pattern, 3 counters are taken away each time");
- providing students with number sequences (e.g., 1, 3, 5, 7, . . .), and asking them to represent the sequence, using concrete materials (e.g., counters, square tiles);

- providing many opportunities to create the same growing or shrinking pattern, using different materials, and to compare the various patterns created;
- providing opportunities to solve problems that involve finding a pattern (e.g., "How many wheels will a bicycle manufacturer need to make 1 bicycle? . . . 2 bicycles? . . . 3 bicycles? . . . 5 bicycles? . . . 10 bicycles?");
- providing opportunities to solve problems that involve growing or shrinking patterns – for example, "In a tile pattern on a wall, the first row of tiles has 12 tiles, the second row has 10 tiles, and the third row has 8 tiles. How many tiles are there in the fourth row? In the sixth row?";
- providing opportunities to make predictions based on patterns (e.g., record the times for sunsets or sunrises in the past 5 days, and then predict times for the next 3 days);
- exploring number patterns, using a calculator (e.g., the pattern created by repeatedly adding 75);
- playing games, such as Guess My Rule, in which students try to determine the rule used by classmates to create a geometric or numeric pattern;
- exploring patterns in a hundreds chart, in addition and multiplication charts, and on number lines (e.g., cover several numbers in a hundreds chart, and have students determine the covered numbers by looking at patterns in the hundreds chart).

Expressions and Equality

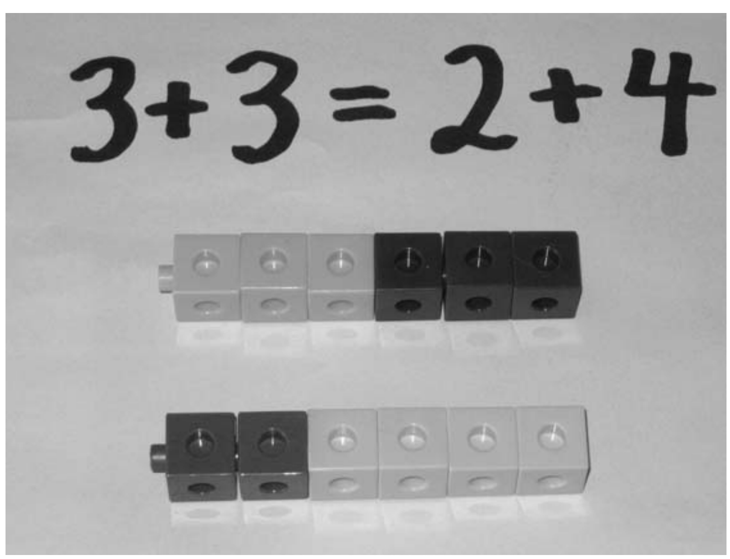
The kind of mathematical thinking that can provide a foundation for learning algebra must be developed over an extended period of time, starting in the early elementary grades.

(Carpenter, Loef Franke, & Levi, 2003, p.1)

Overview

In the primary grades, students observe regularities in the ways that numbers work, and they begin to develop an understanding of the properties of numbers and operations. This understanding provides students with a foundation for the development of more complex and abstract algebraic concepts in higher grades.

Many adults associate algebra with symbol manipulation – solving complicated equations and simplifying algebraic expressions. Their school experiences with algebra may have focused on learning rules and procedures, rather than on the number relationships and operations. By helping young students recognize the properties of numbers and operations, teachers can help them develop a conceptual basis for learning and making sense of algebraic ideas. Throughout the primary grades, students need many opportunities to discuss and to make generalizations about numbers and number properties presented in concrete form. These experiences help them analyse number relationships abstractly in later grades.



The following key points can be made about expressions and equality in the primary grades:

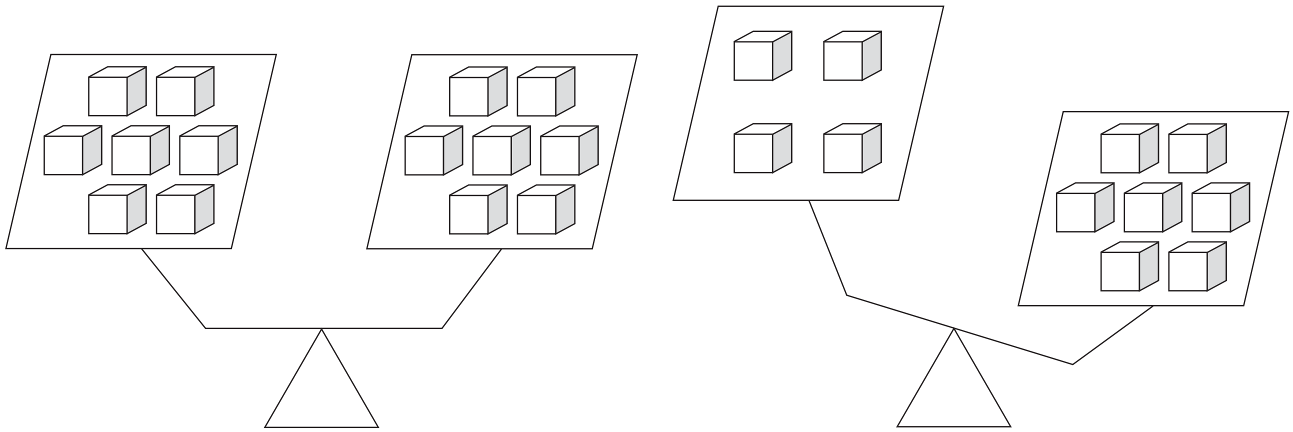
- Equations express the equality between quantities.
- Variables are used to represent unknown quantities, to represent quantities that vary, and to generalize number properties.

Equations

Equations are mathematical statements that have equivalent expressions on both sides of an equal sign. Developing an understanding of equations and of the appropriate use of the equal sign is critical for expressing generalizations about numbers and for developing algebraic reasoning (Carpenter, Loef Franke, & Levi, 2003). Interpreting an equation depends on an understanding of the equal sign as a symbol that separates two expressions that are the same or of equal value ($5 = 5$, $5 = 3 + 2$, $3 + 2 = 4 + 1$). An understanding of the equal sign is essential in higher grades, where students manipulate numbers in equations to solve problems. If these students lack a solid understanding of equality, they become dependent on memorizing algebraic rules and procedures that may have little meaning or relevance to them.

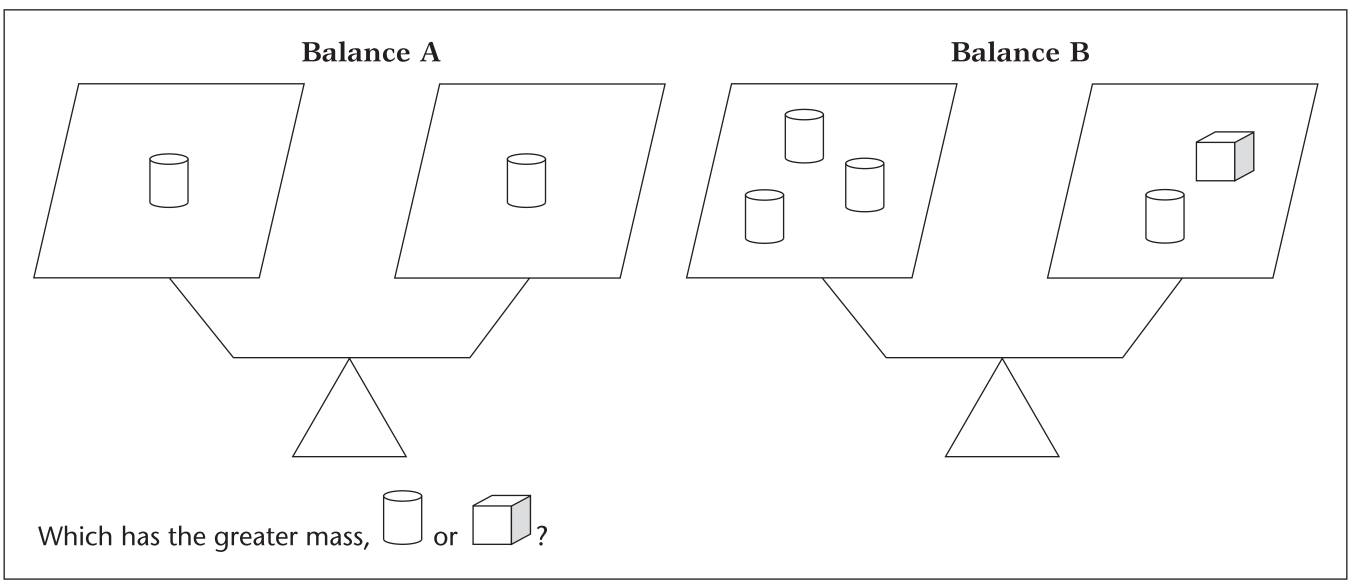
In the primary grades, students often misinterpret the equal sign – they consider it to be a signal to perform an operation (addition, subtraction, multiplication, division), rather than a symbol that expresses the equality between two quantities (Kilpatrick, Swafford, and Findell, 2001). For example, in solving $4 + 5 = \_\_ + 2$, students often incorrectly answer "9", because they think that the equal sign is an indication to perform an operation (e.g., $4 + 5$) and that the number following the sign needs to be the answer to that operation (Falkner, Levi, & Carpenter, 1999). To help students interpret the equal sign correctly, teachers should use phrases such as "the same as" for the equal sign, rather than expressions such as "gives an answer of" or "makes". For example, in reading the equation $4 + 5 = 9$, teachers should say, "Four plus five is the same as nine," rather than "Four plus five gives an answer of nine." By focusing on the equality (sameness) of both sides of the equation, teachers help students understand that the equal sign represents a relationship and that it is not a signal to perform an operation.

Children need many experiences in recognizing, defining, creating, and maintaining equality (Taylor-Cox, 2003). Experiences with a balance provide a concrete way for students to explore ideas about equality and inequality. For example, students can observe that the sides are balanced when they contain the same number of cubes and that they are not balanced when one side contains more cubes than the other.



Students also discover that sides that are balanced remain balanced when the same number of cubes is added to or taken away from both sides. Ideas about maintaining equality are important concepts for students to understand. The focus in experiments with a balance is not on the differences between the two sides, but rather on what is needed to maintain balance or equality between the two sides.

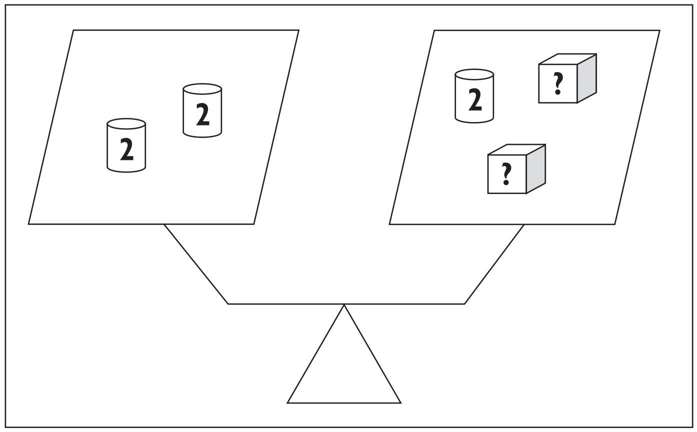
Activities involving balances, such as the following, can also challenge students to identify the value of objects in a situation of equality.



Students might use logic to solve the preceding problem.

- Balance A indicates that both cylinders are equal.
- Balance B indicates that three cylinders are equal to one cylinder and one cube.
- One cylinder can be removed from both sides in balance B, and the sides will remain balanced.
- Two cylinders are equal to one cube. Therefore, a cube has a greater mass than a cylinder.

Problems involving balances can also involve finding unknown values. For example, cylinders could be assigned a value of 2, and cubes could be given an unknown value.



In order to find the value of a cube in the preceding example, students might reason in the following way:

- A cylinder could be removed from both sides, and the sides would remain balanced.
- A cylinder (2) is equal to two cubes.
- A cube is worth 1.

Seeing equations presented in a variety of ways gives students further opportunities to think about the meaning of equality. Most often, equations are presented in the form 5 + 2 = 7, where the operation sign is on the left side of the equation. Examining equations such as 7 = 5 + 2 dispels the notion that the operation sign must be on the left side of the equation. Discussions about open-number sentences (e.g., 4 + 3 = __ + 5, 5 + 2 = 2 + __) also provide opportunities for students to think about the equality of expressions on both sides of an equation.

Having students determine whether equations are true or false is an instructional strategy that focuses on the meaning of equality (Carpenter, Loef Franke, & Levi, 2003). For example, students might be asked to explain whether number sentences such as the following are true or false.

3 + 5 = 8
8 = 3 + 5
8 = 8
3 + 5 = 3 + 5
3 + 5 = 5 + 3
3 + 5 = 4 + 4

Students may think that equations presented in unfamiliar forms (e.g., 8 = 3 + 5, rather than 3 + 5 = 8) are incorrect. Opportunities to model and discuss these equations, using concrete materials, provide rich learning experiences that help students understand important mathematical concepts.

Variables

A variable is a letter or symbol that is used to represent an unknown quantity (e.g., ■ + 4 = 12), a value that can vary (e.g., 4 + ○ = □), or a generalized number property (e.g., $a + b = b + a$). In the primary grades, geometric shapes, rather than letters, are often used as variables, since in the early stages of learning to read, students identify letters as sounds, rather than as quantities.

Variables representing unknown quantities. Variables can be used to represent unknown quantities that have a fixed value. In the following equations, each variable represents one and only one value.

□ + 1 = 7
12 = △ + 5

Number sentences in which variables represent unknown quantities are often used in the primary grades when students learn number facts. Providing real-life problems, and using number sentences to model these situations, help students understand how variables can represent unknown numbers. The following example shows how a problem situation might be expressed as a number sentence with a variable.

Troy is creating a picture book with 10 pictures. He has drawn 6 pictures. How many more pictures does Troy have to draw?

6 + △ = 10

Students can also discuss how to use different number sentences to express the same problem situations. For example, they could represent the preceding problem in the following ways:

△ + 6 = 10
10 = △ + 6
10 – 6 = △
10 – △ = 6

Variables representing quantities that vary. Variables can represent quantities that have no fixed value. In these equations, the value of one variable depends on the value of the other. In the following example, substituting any number for the triangle affects the value of the square.

△ + □ = 7

These types of equations are commonly used in activities in which students find number combinations for a given value. For example, students might use two colours of interlocking cubes to find number combinations for 6 (e.g., 0 and 6, 1 and 5, 2 and 4, 3 and 3, and so on).

Variables used to generalize properties. In the primary grades, students begin to understand the regularities in number operations, and begin to make generalizations about number properties. They also learn that number expressions with variables can be used to represent these properties. The following chart outlines number properties learned by students in the primary grades, and shows how variables might be used to express each property.

Number Property	**Generalized Expression**
Addition and subtraction are inverse operations (e.g., since 5 + 6 = 11, then 11 – 6 = 5).	If □ + ○ = ☆ , then ☆ – ○ = □
Multiplication and division are inverse operations (e.g., since 3 x 7 = 21, then 21 ÷ 7 = 3).	If □ x ○ = ☆ , then ☆ ÷ ○ = □
Adding 0 to or subtracting 0 from any number does not change the number's value (e.g., 6 + 0 = 6; 7 – 0 = 7).	□ + 0 = □ □ – 0 = □
Multiplying or dividing a number by 1 does not change the number's value (e.g., 8 x 1 = 8, 7 ÷ 1 = 7).	□ x 1 = □ □ ÷ 1 = □
Any number subtracted from itself results in 0 (e.g., 9 – 9 = 0).	□ – □ = 0
Any number divided by itself results in 1 (e.g., 8 ÷ 8 = 1).	□ ÷ □ = 1
The product of any number and 0 is 0 (e.g., 4 x 0 = 0)	□ x 0 = 0
Numbers can be added in any order without affecting the sum (e.g., 2 + 58 = 58 + 2).	□ + ○ = ○ + □
In addition, the numbers being added can be regrouped in any way without changing the sum. For example, (13 + 4) + 6 = 13 + (4 + 6).	(□ + ○) + ☆ = □ + (○ + ☆)

Research (Carpenter, Loef Franke, & Levi, 2003) indicates that primary students have an implicit knowledge of basic number properties. However, they may not be able to generalize this knowledge. Instruction must take advantage of this implicit knowledge and provide experiences that allow students to make generalizations about number relationships. Rather than explaining arbitrary rules (e.g., "When zero is added to a number, the number stays the same"), teachers should involve students in investigations using concrete materials that help them think about and generalize properties in meaningful ways. While discussing and reflecting on discoveries made during these investigations, students can examine how number expressions with variables, such as those given in the chart on the preceding page, can express generalizations about number properties.

Primary students may have difficulties in understanding concepts about 0 and in performing operations involving 0. Generally, students do not include 0 when counting, and rarely do they refer to 0 as a quantity, except in situations in which all of a set of objects have been taken away. Students need many opportunities to investigate the meaning of 0 in order to make generalizations about operations involving 0.

Early misconceptions about 0 are potentially detrimental to students' understanding of number properties and subsequent development of algebraic thinking (Anthony & Walshaw, 2004). Discussing whether given number expressions involving 0 are true or false provides students with an opportunity to investigate and describe number properties (Carpenter, Loef Franke, & Levi, 2003). For example, teachers might present number expressions such as the following, and ask students to explain whether each number sentence is true or false.

$9 + 5 = 14$
$9 + 5 = 14 + 0$
$9 + 5 = 0 + 14$
$9 + 5 = 13 + 1$

Through investigations and rich discussions of number properties, students are able to make generalizations for themselves and to develop concepts that provide a foundation for more formalized algebraic thinking.

Characteristics of Student Learning and Instructional Strategies by Grade

Kindergarten

Characteristics of Student Learning

In general, students in Kindergarten:

- begin to use quantitative terms (e.g., *the same as*, *more than*, and *fewer than*) to compare quantities;
- recognize that a numeral represents the number of objects in a set;
- recognize that sets of objects can be decomposed into groups (e.g., 6 toy cars can be separated into a group of 2 cars and a group of 4 cars);
- know that a set of objects gets larger or smaller when objects are added or subtracted;
- begin to recognize that 0 signifies an empty set.

Instructional Strategies

Students in Kindergarten benefit from the following instructional strategies:

- having them explore and discuss sets that have the same number of objects (e.g., a set of 5 plastic bugs has the same number of objects as a set of 5 toy cars);
- providing opportunities to create sets that are the same as, more than, or fewer than a given quantity of 10 or fewer items, using concrete materials;
- having them show different arrangements of the same quantity, using dot plates, five frames, ten frames, and other concrete materials.

Grade 1

Characteristics of Student Learning

In general, students in Grade 1:

- use quantitative terms (e.g., *the same as*, *equal to*, *more than*, and *less than*) to compare quantities;
- recognize that a numeral represents the number of objects in a set;
- recognize that numbers can be decomposed into parts, and know that different combinations have the same value (e.g., 19 can be separated into 14 and 5, 13 and 6, 12 and 7, and so on);
- know that a set of objects gets larger or smaller when objects are added or subtracted;

- recognize that 0 signifies an empty set;
- understand that the equal sign means "the same as", and that the two sides of an equation have the same value (e.g., 10 and 7 + 3 have the same value in 10 = 7 + 3);
- understand the concept of equality (e.g., use a balance to show equal quantities).

Instructional Strategies

Students in Grade 1 benefit from the following instructional strategies:

- providing opportunities to explore concepts related to equality and inequality. For example, students might investigate how adding the same quantity to (or removing the same quantity from) both sides of a balance maintains equality;
- providing many experiences in demonstrating equality, using concrete materials (e.g., create different number combinations for 8, such as 1 and 7, 2 and 6, 4 and 4, and so on, using two colours of interlocking cubes);
- providing experiences in exploring the concept of 0 (e.g., identify objects of which there are 0 in the classroom; take away objects from a set to show the concept of 0);
- prompting students to make generalizations about number properties (e.g., "If you add 0 to a number, the number stays the same");
- providing opportunities to view and discuss equations written in a variety of formats (e.g., 3 = 3, 4 = 3 + 1, 4 + 1 = 5).

Grade 2

Characteristics of Student Learning

In general, students in Grade 2:

- understand that different combinations of numbers can represent the same value (e.g., 15 + 3 = 4 + 14);
- understand and apply the commutative property of addition (i.e., the numbers in an addition expression can be added in any order, as in 4 + 6 = 6 + 4);
- generalize the properties of 0 in addition and subtraction (e.g., when you add 0 to a number, the number does not change; when you subtract 0 from a number, the number does not change);
- recognize that a variable (e.g., a shape such as a square or circle) in an equation represents an unknown quantity;
- solve problems involving an unknown quantity in an addition or a subtraction situation;
- find the missing number in an equation involving addition or subtraction.

Instructional Strategies

Students in Grade 2 benefit from the following instructional strategies:

- providing opportunities to explore concepts related to equality and inequality. For example, students might investigate how adding the same quantity to (or removing the same quantity from) both sides of a balance maintains equality;
- providing many experiences in demonstrating equality, using concrete materials (e.g., show that a combination of 18 counters and 2 counters is equivalent to a combination of 13 counters and 7 counters);
- providing opportunities to discuss the meaning of equations (e.g., have students explain whether given number sentences, such as $2 + 3 = 5 + 1$ and $7 - 7 = 0 + 7$, are true or false);
- providing opportunities to view and discuss equations written in a variety of formats (e.g., $8 = 5 + 3$, $4 + 2 = 2 + 4$, $12 + 3 + 3 = 15 + 3$);
- discussing and demonstrating number properties (e.g., use counters to demonstrate the commutative property of addition);
- providing experiences in exploring the concept of 0 (e.g., subtract objects from a set to show the concept of 0; use a calculator to perform operations involving 0, such as $6 - 6$);
- providing opportunities to solve problems involving unknown quantities in addition and subtraction situations;
- providing opportunities to find the missing number in equations involving addition and subtraction to 18 (e.g., find the missing number in $6 + 3 = 4 + \square$, using counters).

Grade 3

Characteristics of Student Learning

In general, students in Grade 3:

- understand and apply the inverse relationship between addition and subtraction (e.g., if $3 + 9 = 12$, then $12 - 9 = 3$);
- understand and apply the properties of 0 and 1 in multiplication (i.e., any number multiplied by 0 equals 0; any number multiplied by 1 equals the original number);
- understand and apply the commutative property of multiplication (e.g., $2 \times 4 = 4 \times 2$);
- recognize that a variable (e.g., a shape such as a square or circle) in an equation represents an unknown quantity;

- solve problems involving an unknown quantity in an addition or subtraction situation;
- find the missing number in an equation involving the addition or subtraction of one- and two-digit numbers.

Instructional Strategies

Students in Grade 3 benefit from the following instructional strategies:

- providing many experiences in demonstrating equality, using concrete materials (e.g., show that a 3×4 array of tiles is equal to a 2×6 array of tiles);
- providing opportunities to view and discuss equations written in a variety of formats (e.g., $19 = 12 + 7$, $41 + 2 = 2 + 41$, $4 + 23 = 4 + 20 + 3$);
- having them generate a variety of equations for a given number (e.g., $8 = 2 + 2 + 2 + 2$, $8 = 4 \times 2$, $8 = 2 \times 4$, $8 = 8 \times 1$, and so on);
- discussing and demonstrating number properties (e.g., use counters to demonstrate the associative property of addition);
- providing experiences in using the associative property to facilitate mental computations with whole numbers (e.g., to find $5 + 4 + 16$, add $4 + 16$ to get 20, and then add $5 + 20$);
- providing many experiences in exploring the properties of 0 and 1 in multiplication (e.g., use counters to show that any number multiplied by 0 equals 0; use a calculator to show that any number multiplied by 1 equals the original number);
- providing opportunities to solve problems involving unknown quantities in addition and subtraction situations;
- providing opportunities to find the missing number in equations involving the addition and subtraction of one- and two-digit numbers (e.g., find the missing number in $26 - 3 = 12 + \square$, using concrete materials).

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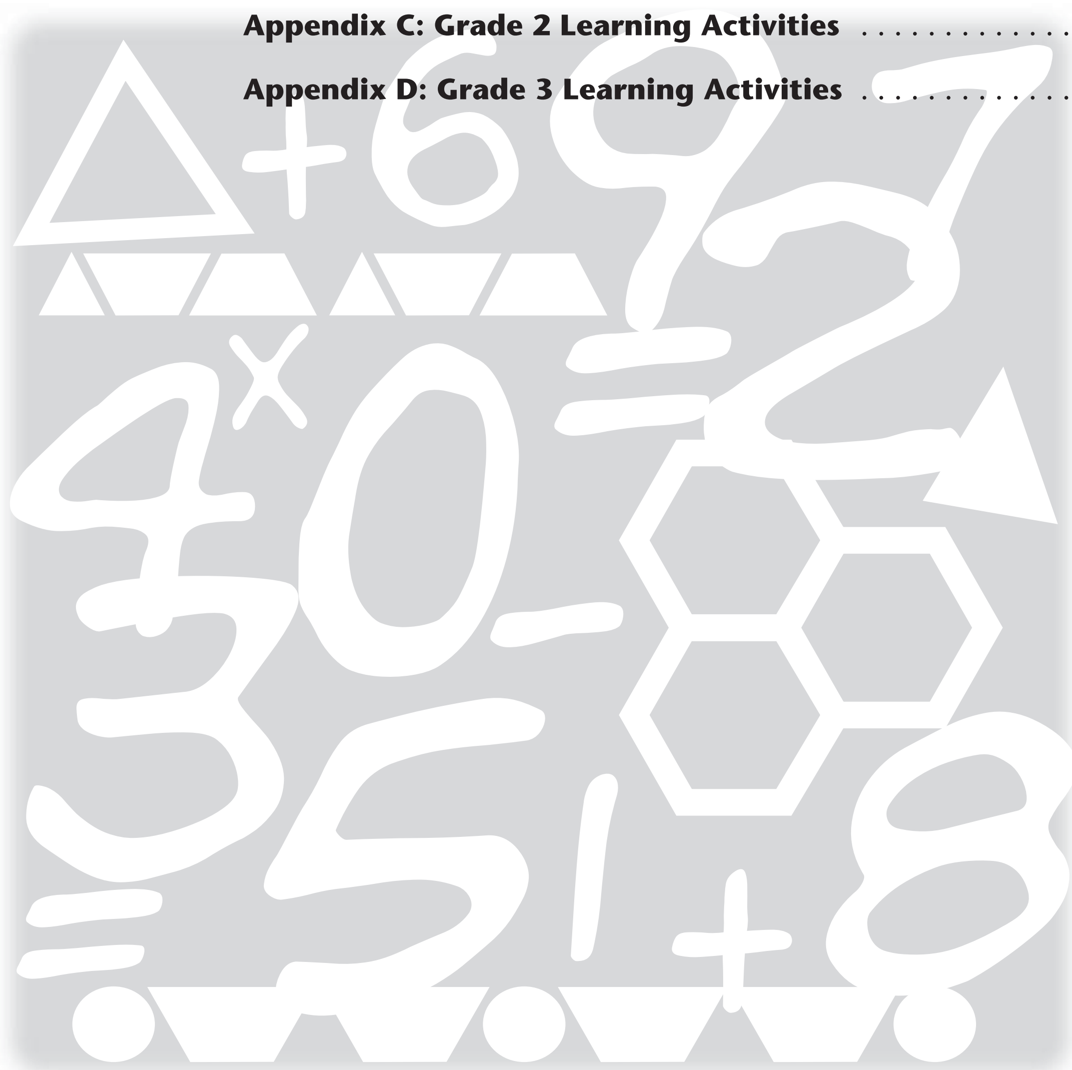
Learning Activities for Patterning and Algebra

Contents



Introduction

The following four appendices (Appendices A to D) include learning activities that are practical applications of the big ideas in Patterning and Algebra for Kindergarten to Grade 3, respectively. For each grade, one activity is included for each of the big ideas: patterns and relationships, and expressions and equality. These activities do not address all the key concepts for each big idea, since the big ideas cannot be fully addressed in one activity. The learning activities provide a starting point for classroom instruction related to the big ideas; however, students need multiple experiences throughout the school year to build an understanding of each big idea.

The learning activities are organized as follows:

- **CURRICULUM EXPECTATIONS:** The curriculum expectations are indicated for each learning activity.
- **MATERIALS:** A materials list is included for the main task in each learning activity. (The learning connections have their own materials lists.)
- **ABOUT THE MATH:** Background mathematical information that connects the learning activity to the big idea is provided. In some instances, reference is made to some of the important learning that should precede the activity.
- **GETTING STARTED:** This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or task.
- **WORKING ON IT:** In this part, students work on a mathematical task, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.
- **REFLECTING AND CONNECTING:** This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.
- **ADAPTATIONS/EXTENSIONS:** These are suggestions for ways to meet the needs of all learners in the classroom.
- **MATH LANGUAGE:** Vocabulary that is important to the learning activity and to the concepts presented is included under this heading.
- **ASSESSMENT:** This section provides guidance to teachers on assessing students' understanding of mathematical concepts related to the big ideas.

- **HOME CONNECTION:** This section is addressed to parents or guardians, and includes a task for students to do at home that is connected to the mathematical focus of the learning activity.
- **LEARNING CONNECTIONS:** These are suggestions for follow-up activities that either consolidate the mathematical focus of the main task or build on other key concepts for the big idea.
- **BLACKLINE MASTERS:** These pages are referred to and used throughout the activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities in Appendices A–D demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or an inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;

- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this document provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and to represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experience in many of the learning activities allows students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Accommodations and Modifications

The learning activities in Appendices A–D have been designed for students with a range of learning needs. Instructional and assessment tasks are open-ended, allowing most students to participate fully in learning experiences. In some cases, individual students may require *accommodations* and/or *modifications*, in accordance with their Individual Education Plan (IEP), to support their participation in learning activities.

The term *accommodations* is used to refer to the special teaching and assessment strategies, human supports, and/or individualized equipment required to enable a student to learn and to demonstrate learning. Accommodations do not alter the provincial curriculum expectations for the grade.

Modifications are changes made in the age-appropriate grade-level expectations for a subject … in order to meet a student's learning needs. These changes may involve developing expectations that reflect knowledge and skills required in the curriculum for a different grade level and/or increasing or decreasing the number and/or complexity of the regular grade-level curriculum expectations.

(Ontario Ministry of Education, 2004, pp. 25–26)

Providing Accommodations

Students may require accommodations, including special strategies, support, and/or equipment to allow them to participate in learning activities. There are three types of accommodations:

- *Instructional accommodations* are adjustments in teaching strategies, including styles of presentation, methods of organization, or the use of technology or multimedia.
- *Environmental accommodations* are supports or changes that the student may require in the physical environment of the classroom and/or the school, such as preferential seating or special lighting.
- *Assessment accommodations* are adjustments in assessment activities and methods that enable the student to demonstrate learning, such as allowing additional time to complete tasks or permitting oral responses to test questions.

Some of the ways in which teachers can provide accommodations with respect to mathematics learning activities are listed in the following chart.

Instructional Accommodations

- Vary instructional strategies, using different manipulatives, examples, and visuals (e.g., concrete materials, pictures, diagrams) as necessary to aid understanding.
- Rephrase information and instructions to make them simpler and clearer.
- Use non-verbal signals and gesture cues to convey information.
- Teach mathematical vocabulary explicitly.
- Have students work with a peer.
- Structure activities by breaking them into smaller steps.
- Model concepts using concrete materials, and encourage students to use them when learning concepts or working on problems.
- Have students use calculators and/or addition and multiplication grids for computations.
- Format worksheets so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Encourage students to use graphic organizers and graph paper to organize ideas and written work.
- Provide augmentative and alternative communications systems.
- Provide assistive technology, such as text-to-speech software.
- Provide time-management aids (e.g., checklists).
- Encourage students to verbalize as they work on mathematics problems.
- Provide access to computers.
- Reduce the number of tasks to be completed.
- Provide extra time to complete tasks.

Environmental Accommodations

- Provide an alternative workspace.
- Seat students strategically (e.g., near the front of the room; close to the teacher in group settings; with a classmate who can help them).
- Reduce visual distractions.
- Minimize background noise.
- Provide a quiet setting.
- Provide headphones to reduce audio distractions.
- Provide special lighting.
- Provide assistive devices or adaptive equipment.

continued

Assessment Accommodations

- Have students demonstrate understanding using concrete materials or orally rather than in written form.
- Have students record oral responses on audiotape.
- Have students' responses on written tasks recorded by a scribe.
- Provide assistive technology, such as speech-to-text software.
- Provide an alternative setting.
- Provide assistive devices or adaptive equipment.
- Provide augmentative and alternative communications systems.
- Format tests so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Provide access to computers.
- Provide access to calculators and/or addition and multiplication grids.
- Provide visual cues (e.g., posters).
- Provide extra time to complete problems or tasks or answer questions.
- Reduce the number of tasks used to assess a concept or skill.

Modifying Curriculum Expectations

Students who have an IEP may require modified expectations, which differ from the regular grade-level curriculum expectations. When developing modified expectations, teachers make important decisions regarding the concepts and skills that students need to learn.

Most of the learning activities in this document can be adapted for students who require modified expectations. The chart on the opposite page provides examples of how a teacher could deliver learning activities that incorporate individual students' modified expectations.

Modified Program	What It Means	Example
Modified learning expectations, *same* activity, *same* materials	The student with modified expectations works on the same or a similar activity, using the same materials.	The learning activity involves identifying and describing growing and shrinking patterns on a number line. Students with modified expectations identify and describe repeating patterns on a number line.
Modified learning expectations, *same* activity, *different* materials	The student with modified expectations engages in the same activity, but uses different materials that enable him or her to remain an equal participant in the activity.	The activity involves determining the missing two-digit number in addition and subtraction equations, using a calculator. Students with modified expectations determine the missing one-digit number in addition and subtraction equations, using counters.
Modified learning expectations, *different* activity, *different* materials	Students with modified expectations participate in different activities.	Students with modified expectations work on patterning activities that reflect their learning expectations, using a variety of concrete materials.

(Adapted from *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6, 2005*, p. 119.)

It is important to note that some students may require both accommodations and modified expectations.

A Special Note About Kindergarten

The Kindergarten years represent a two-year continuum for those children who attend both Junior Kindergarten and Senior Kindergarten. In many classrooms, Junior Kindergarten and Senior Kindergarten students work together in multi-age groups. Therefore, it is important to assess and consider students' level of development of early mathematical understandings before planning any math activities. Many of the Patterning and Algebra learning activities are multilevel and can be used with both age groups. In some cases, suggestions are made for adapting an activity for students in Junior Kindergarten.

Often, teachers in a multi-age classroom have the Senior Kindergarten students complete a small-group or independent follow-up activity after modelling or demonstration is done for the whole class. When invited, many Junior Kindergarten students will join in the activity, even though they are not required to participate. This willingness to learn can give teachers a greater understanding of students' level of mathematical knowledge and skills. Although teachers will have different expectations for younger students, sometimes the level of understanding that Junior Kindergarten students demonstrate surprises teachers. Providing instruction that meets the unique needs of each student helps to support further learning.

A. Kindergarten Learning Activities

Appendix Contents

Colourful Caterpillars

BIG IDEA **Patterns and Relationships**

CURRICULUM EXPECTATIONS

Students will:

- identify, extend, reproduce, and create repeating patterns through investigation, using a variety of materials *(e.g., attribute materials, pattern blocks, a hundreds chart, toys, bottle tops, buttons, toothpicks)* and actions *(e.g., physical actions such as clapping, jumping, tapping).*

MATERIALS

- overhead colour tiles
- blank overhead transparency
- overhead projector
- overhead marker
- containers of blue, yellow, green, and red colour tiles (to be shared by students)
- a large quantity of blue, yellow, green, and red squares cut from construction paper (same size as colour tiles)
- crayons
- 8 ½ in. x 11 in. sheets of paper (1 per student)
- several glue sticks (to be shared by students)
- PRK.BLM1: Patterns at Home

ABOUT THE MATH

Kindergarten students require many opportunities to explore repeating patterns. Initially, students identify, extend, and create simple repeating patterns involving obvious attributes, such as colour (e.g., red, blue, red, blue, red, blue, red, blue, . . .). With experience and guidance from teachers, students begin to investigate more complex patterns. The complexity of a pattern is determined by the size of its core (i.e., the number of elements in the part of the pattern that repeats), and by the number of changing attributes (e.g., colour, shape, size).

Investigating patterns helps students make generalizations, develop reasoning strategies, and build a foundation for more complex algebraic thinking. An important way in which teachers can promote a greater understanding of patterns and relationships is to encourage students to work with increasingly complex patterns.

In the following activity, students create patterns, using colour tiles, and record the patterns, using coloured paper. By examining their own patterns and those created by classmates, students have an opportunity to investigate patterns of varying degrees of complexity.

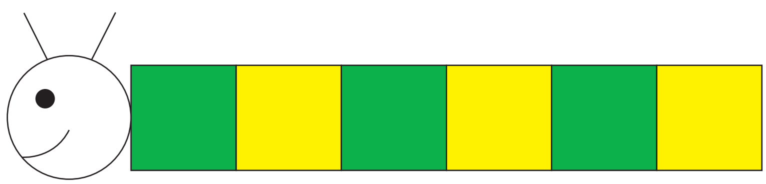
GETTING STARTED

Have students stand in a circle. Begin performing a simple pattern that involves repeating actions (e.g., repeatedly clapping your hands and touching your knees). Encourage students to join in performing the pattern. Continue the pattern until all students are participating. Have students describe the pattern (e.g., clap your hands, touch your knees, clap your hands, touch your knees, clap your hands, touch your knees).

Continue the activity by modelling other action patterns (e.g., repeatedly touching your head and touching your shoulders), and invite students to join in. Have students describe each pattern. Patterns may be teacher-directed or student-directed, depending on the abilities of the students.

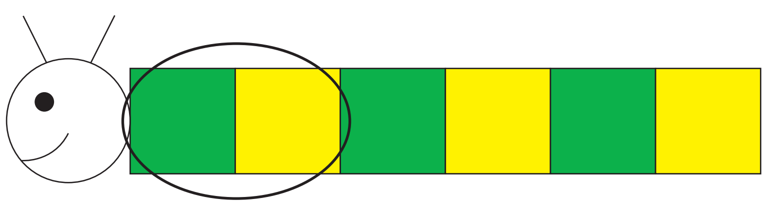
WORKING ON IT

Using overhead colour tiles, display a simple repeating pattern (e.g., green, yellow, green, yellow, green, yellow) on a blank transparency on the overhead projector. Draw a circle at one end of the pattern and add eyes and antennae for the caterpillar.



Ask: “What colour comes next in our caterpillar? How do you know?” Have different students take turns adding colour tiles to the caterpillar. Point to each colour tile and have the class read the pattern aloud (e.g., “green, yellow, green, yellow, green, yellow, … ”).

Have students identify the repeating part of the pattern. Mark the pattern core, using an overhead pen.



Show the students other patterns, using colour tiles. Ask students to describe and extend each pattern. Introduce more complex patterns by using three or four colours (e.g., red, yellow, blue, red, yellow, blue, red, yellow, blue) and/or by increasing the complexity of the pattern core (e.g., blue, blue, green, blue, blue, green, blue, blue, green).

Show students how to make their own caterpillar patterns. Create a pattern, using colour tiles, and ask students to describe the pattern. Next, demonstrate how to select construction-paper squares that match the colour tiles in the pattern, and glue the paper squares onto a sheet of paper. Draw a circle at one end of the pattern to create a head for the caterpillar. Explain to students that they may add antennae and eyes to the circle after they have completed their patterns.

As students create colour-tile caterpillars, ask them to point to and read the pattern aloud (e.g., "red, red, yellow, yellow, red, red, yellow, yellow, red, red, yellow, yellow"). Have students identify the colour that comes next in the pattern, and have them extend their patterns.

Students might use different strategies for creating their paper patterns. Some students might create a pattern with colour tiles directly on the paper, and then remove one colour tile at a time, gluing a paper square in its place. Other students might place their paper near their colour-tile pattern, and arrange the paper pattern by matching each paper square with a colour tile. Some students might internalize the pattern and not need to refer directly to their colour-tile pattern.

Some students may require assistance with recording their patterns. For example, you might observe that some students create paper patterns that do not match their colour-tile patterns. Have these students point to and read their colour-tile pattern, and then ask them to show the same pattern on their paper.

REFLECTING AND CONNECTING

Gather the students in a large circle, and have them place their completed caterpillar patterns on the floor in front of them. Ask a student who made a simple two-colour pattern to show his or her work to the class, and have the students read the pattern aloud. Ask: "What comes next in this pattern? How do you know?" Repeat by having other students explain their patterns to the class. Attempt to include a variety of patterns (e.g., patterns with different numbers of colours, patterns with different core structures).

Create a bulletin board display called "Caterpillar Patterns" to show the students' work.

ADAPTATIONS/EXTENSIONS

Students, especially Junior Kindergarten students, may have difficulty creating square-paper patterns that match a colour-tile pattern. Have these students create either colour-tile or square-paper patterns.

To increase the level of pattern complexity, have students include attributes such as shape or size in their patterns. For example, students could use pattern blocks or attribute blocks to create caterpillar patterns.

MATH LANGUAGE

- pattern
- repeating pattern
- extend
- create

ASSESSMENT

Observe students as they create and describe their patterns.

- *Are students able to create patterns?*
- *What strategies do they use to create and record patterns?*
- *Can students describe and extend simple patterns?*

HOME CONNECTION

Send home PRK.BLM1: Patterns at Home. This letter encourages parents and students to explore patterns, using home materials. Parents are asked to have their child draw a picture of a pattern created at home. Provide an opportunity for students to share their work with the class.

LEARNING CONNECTION 1

People Patterns

Kindergarten students enjoy being part of a pattern. People patterns help students recognize repetitions that occur within a pattern. People patterns should be introduced early in the year and can be used frequently, becoming more complex as the year progresses. To introduce a people pattern, have students sit or stand in a circle, so that they can watch the pattern as it develops. Begin with a simple pattern (e.g., stand, sit, stand, sit, stand, sit). Establish the pattern by having the first few students get into their appropriate positions. Ask students to read the pattern aloud (e.g., "stand, sit, stand, sit, stand, sit"). As the rest of the students in the circle take appropriate sitting or standing positions, have all the students continue to say aloud the position of each student.

Repeat the activity by asking students to suggest other people patterns (e.g., girl-girl-boy, facing inward and facing outward). Together with the students, create the people patterns. To assess whether students recognize patterns, ask individuals to tell students in the circle which position to take.

As the class creates people patterns, ask questions such as the following:

- "How did you know what you need to do in the pattern?"
- "How can you tell what comes next in the pattern?"
- "How would you describe this pattern to someone else?"
- "How does this pattern compare with the last one we did?"

LEARNING CONNECTION 2

What Is the People Pattern?

Ask half of the class to stand in a line in front of the rest of the students. Ask a student to think of a people pattern (e.g., standing-kneeling-kneeling, standing-kneeling-kneeling, standing-kneeling-kneeling) and to whisper a position to each student in the line.

After all students in the line have taken a position, ask the class to identify the people pattern. Discuss how the pattern would continue if more students joined the line.

Repeat the activity, using different lines of students. Ask different students to think of a people pattern and to whisper positions to students in the line. Have the class identify and extend the patterns.

LEARNING CONNECTION 3

A New Dance

Begin performing an action pattern (e.g., a pattern involving a repetition of jump, jump, turn), and invite some students to join in. As those students participate in the action pattern, play a selection of lively music. Stop the music and ask students to describe the actions involved in the dance.

Discuss other actions that might be part of a dance (e.g., clapping, hopping, raising arms in the air, stomping feet). Invite other students to think of actions for a new dance. Play music and invite individuals to start the dance. Encourage students to join in the dance when they recognize the action pattern.

LEARNING CONNECTION 4

Extend My Pattern

Materials

- pattern blocks

Show students a pattern using pattern blocks (e.g., a pattern involving the repetition of red trapezoid, green triangle, green triangle). Show at least three complete repetitions of the pattern core. Ask: "What comes next in the pattern? How do you know?" Discuss and model how the pattern continues.

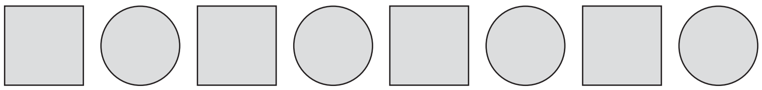
Repeat the activity by showing other patterns using pattern blocks. Create patterns with a level of complexity that is appropriate for the developmental level of the students.

Depending on the readiness of students, the activity might be done by pairs of students. One student creates a simple pattern, and then the partner extends the pattern. Together, pairs of students check that the pattern was created and extended correctly.

Patterns at Home

Dear Parent/Guardian:

In class, we have been learning about repeating patterns, such as the following:



We have been describing, creating, and extending patterns. We have also been identifying the core of the pattern (the part of the pattern that repeats). In the example above, the core is composed of a square and a circle, and the core repeats 4 times.

Patterns can be created in a variety of ways: using different objects (e.g., forks and spoons), different shapes (e.g., circles and squares), different colours (e.g., red and blue), or different sizes (e.g., large and small).

Explore some simple patterns with your child. Create a simple repeating pattern, using two kinds of household objects. For example you might create a pattern such as spoon, spoon, fork, fork, spoon, spoon, fork, fork, spoon, spoon, fork, fork. Ask your child to read the pattern aloud by pointing to and naming each object in the pattern (e.g., "spoon, spoon, fork, fork", and so on). Ask your child to explain what comes next in the pattern, and have him or her extend the pattern, using more of the two kinds of objects chosen.

Next, have your child create a pattern (e.g., blue sock, white sock, blue sock, white sock, blue sock, white sock, and so on). Ask your child to read the pattern aloud, and to extend the pattern by adding a few more objects.

Have your child draw a picture of a pattern you explored together. If the pattern involves colour, ask your child to colour the objects in the picture. Have your child bring the drawing to school, so that it can be shared with the class.

Have fun exploring patterns!

What a Handful!

BIG IDEA **Expressions and Equality**

CURRICULUM EXPECTATIONS

Students will:

- investigate some concepts of quantity through identifying and comparing sets with more, fewer, or the same number of objects *(e.g., find out which of two cups contains more or fewer beans, using counters; investigate the ideas of more, less, and the same, using five and ten frames; compare two sets of objects that have the same number of items, one set having the items spread out, and recognize that both sets have the same quantity [concept of conservation]; recognize that the last count represents the actual number of objects in the set [concept of cardinality]; compare five beans with five blocks, and recognize that the number 5 represents the same quantity regardless of the different materials [concept of abstraction]).*

Note: Although this expectation belongs to the Number Sense and Numeration strand in the Kindergarten program, it reflects concepts about equality that are developed in Kindergarten.

MATERIALS

- containers of approximately 30 to 40 cubes (1 container per pair of students)
- spinners made of EEK.BLM1: "What a Handful!" Spinner, a paper clip, and a pencil (1 spinner per pair of students)
- EEK.BLM2: What a Handful! (1 per student)
- EEK.BLM3: More, Fewer, and Equal (1 per student)

ABOUT THE MATH

Equality is an important concept that students should begin to understand in the primary grades. As early as Kindergarten, students should have opportunities to recognize, describe, and create situations that involve equality (e.g., create equal sets of objects).

In Kindergarten, students learn to compare the number of objects in two sets, using the language of comparison (e.g., *more*, *fewer*, *less*). They also begin to develop concepts about equality – two sets are equal if they contain the same number of objects. Initially, students use informal language to describe equality (e.g., *the same as*, *the same number as*). Teachers can model language such as *equal to* and can encourage students to use this vocabulary.

In the following learning activity, students explore concepts of *more*, *less*, *fewer*, *the same number*, and *equal* by comparing the quantities of sets of cubes.

GETTING STARTED

Have students act out a few situations that demonstrate the meanings of *more*, *less*, *fewer*, and *the same number*. For example, ask two students to go to the sand table, and ask four students to go to the home centre. Ask questions such as the following:

- "Are there more children at the sand table or at the home centre? How do you know?"
- "Are there fewer children at the home centre than at the sand table? How do you know?"
- "Is 2 less than 4? How do you know?"

Provide similar opportunities for students to act out and discuss situations that involve *more*, *less*, *fewer*, and *the same number*. Introduce the term *equal* as another way to say "the same number as".

WORKING ON IT

Demonstrate the following activity:

- Each pair of students needs a container of cubes and a spinner. (Show students how to make a spinner, using a paper clip, a pencil, and EEK.BLM1: "What a Handful!" Spinner.)
- One partner takes a handful of cubes from the container and places the cubes in a pile between the two students.
- The other student spins the spinner. The student creates a second set of cubes according to the word shown on the spinner. For example, if the spinner lands on "More", the student tries to create a set with more cubes than are in the set of his or her partner.
- Students count the cubes in both sets, in order to check whether the second set has more or fewer cubes than the first, or whether the two sets have an equal number of cubes.
- Students take turns as they repeat the activity several times.

As students do the activity, ask questions such as the following:

- "Why do you think there are more (fewer, the same number of) cubes in this set?"
- "How can you check to see which set has more cubes?"
- "Is 5 less than 4? How do you know?"
- "What does 'equal' mean? How do you know if both sets have an equal number of cubes?"

After students have had an opportunity to do the activity several times, provide each student with a copy of EEK. BLM2: What a Handful! Ask students to take a handful of cubes and draw the cubes (using square shapes) on the top hand on the sheet. Next, have students draw cubes on the other three hands to show quantities that represent “More”, “Fewer”, and “Equal”. Invite the students to use actual cubes to help them find quantities of cubes to represent in their drawings.

REFLECTING AND CONNECTING

Gather students to talk about the activity with the spinner. Ask the following questions:

- “How did you decide which set had more cubes?”
- “How did you decide whether both sets had an equal number of cubes?”
- “How could you show your partner that you had the same number/a different number of cubes?”
- “What does ‘equal’ mean? When did you know that you had two sets with an equal number of cubes?”

Invite students to show their work on EEK.BLM2: What a Handful! to the class. Have them explain how they determined the number of cubes to draw on the three hands.

ADAPTATIONS/EXTENSIONS

Students who find the activity too difficult may work with a partner to compare two sets of cubes, without using a spinner (e.g., “This pile has more cubes than this pile”).

Students who need a greater challenge could determine how many more or how many fewer cubes are in one set than in another. They might also determine the number of cubes that need to be added to (or taken from) a set of cubes to make it equal to another set.

MATH LANGUAGE

– more
– less
– fewer
– same number
– equal

ASSESSMENT

During the activity, observe students to assess how well they understand concepts of *more*, *less*, *fewer*, and *equal*. As well, observe their strategies for determining whether one set has more or fewer cubes than another set, or whether both sets have an equal number of cubes. For example, some students might count the cubes in each set and compare the quantities; other students might arrange the cubes in each set in a row and then compare the lengths of the two rows of cubes.

HOME CONNECTION

Send home EEK.BLM3: More, Fewer, and Equal. In this Home Connection activity, students create sets of objects to show ideas about *more*, *less*, and *equal*.

LEARNING CONNECTION 1

Dot Plate Pairs

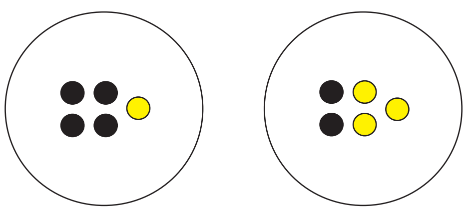
Materials

- EEK.BLM4: Paper Plate Dot Arrangements
- paper plates
- circle stickers or bingo dabbers

Ahead of time, create dot plates, using paper plates, circle stickers or bingo dabbers, and EEK.BLM4: Paper Plate Dot Arrangements.

Provide each pair of students with a collection of dot plates, and have them place the plates in front of them. Instruct students to find pairs of plates that have the same (an equal) number of dots. Students may either take turns or work cooperatively to find pairs of plates.

Encourage students to discuss how pairs of plates show an equal number of dots. For example, 4 black dots and 1 yellow dot are "the same as" or "equal to" 3 yellow dots and 2 black dots.



To challenge students, have them choose a plate and then find plates that have more or fewer dots than the chosen plate.

Variations of the activity include having students match dot plates to dominoes or to number cards.

LEARNING CONNECTION 2

Watermelon Seeds

Materials

- cards cut from EEK.BLM5a–b: Number Dot Cards
- EEK.BLM6: Watermelon Slices
- black cubes (or similar concrete materials, such as beans painted black or black counters)

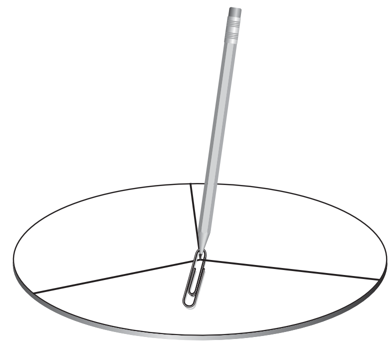
Demonstrate the activity:

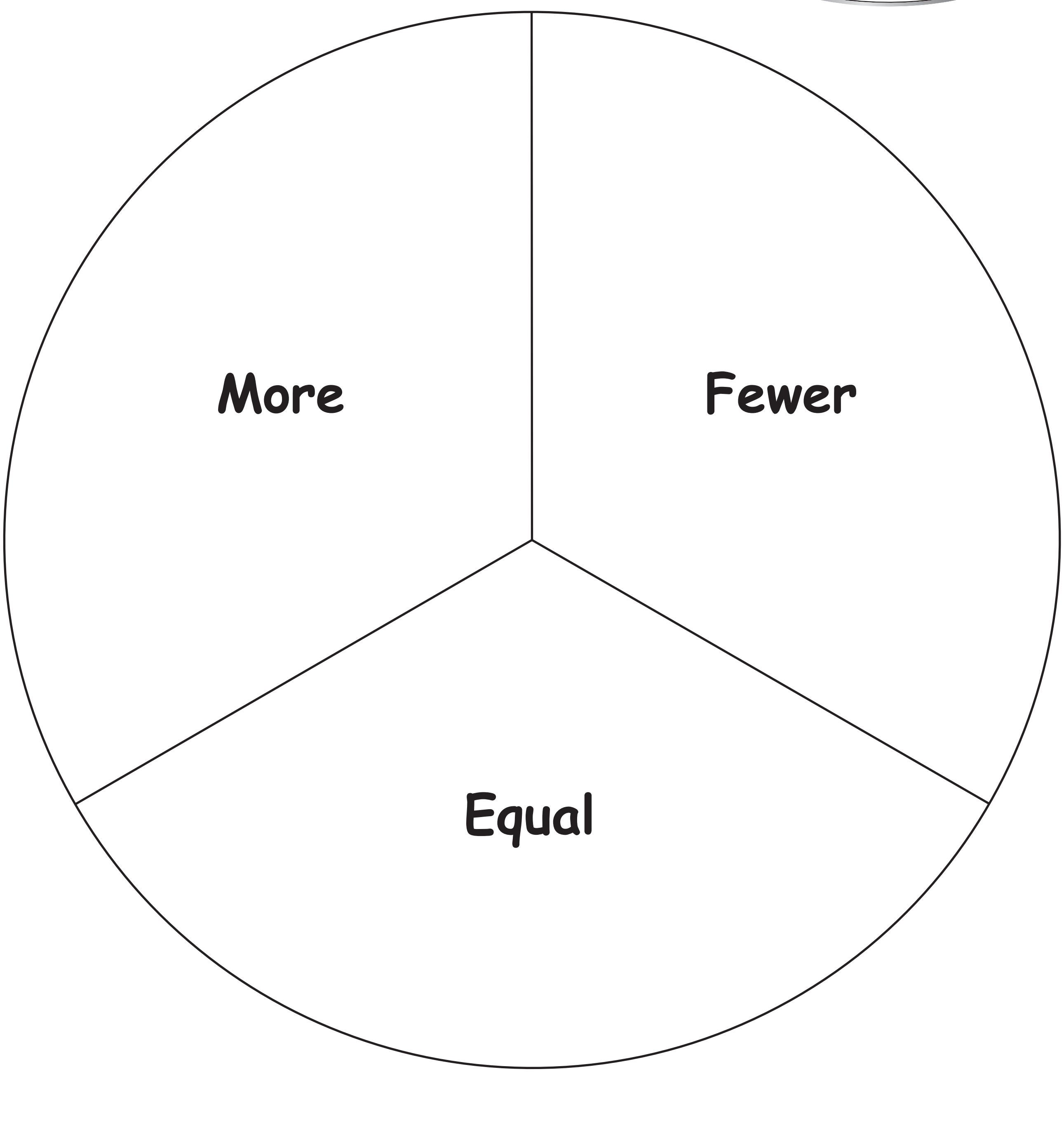
- Students select a number dot card (made from EEK.BLM5a–b: Number Dot Cards).
- Students create three sets of "watermelon seeds" by placing black cubes on EEK.BLM6: Watermelon Slices. The sets show quantities of seeds that are more than, fewer than, and equal to the number represented on the dot card.

Provide students with opportunities to do the activity at a learning centre.

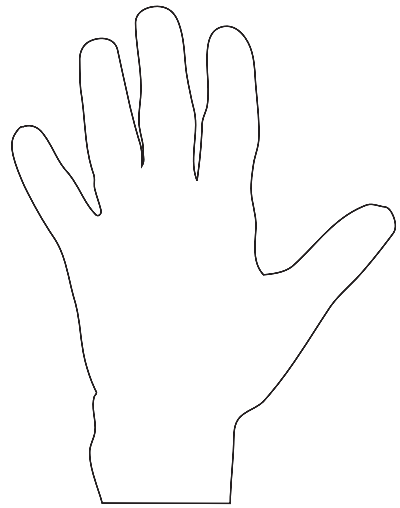
"What a Handful!" Spinner

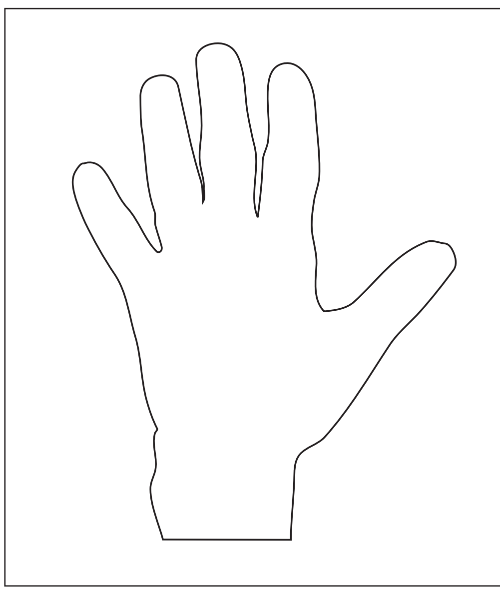
Make a spinner, using this page, a paper clip, and a pencil.



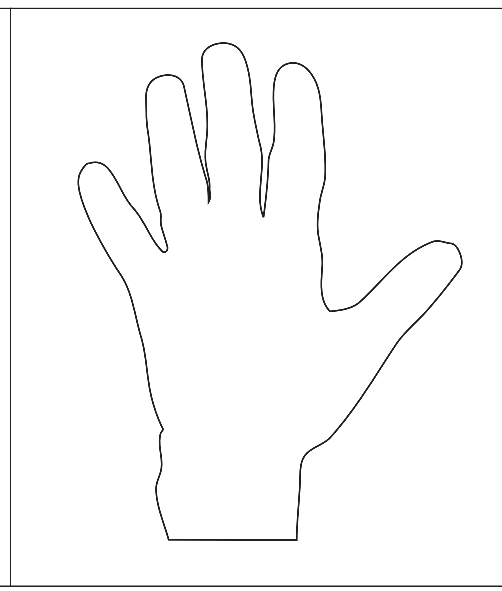


What a Handful!

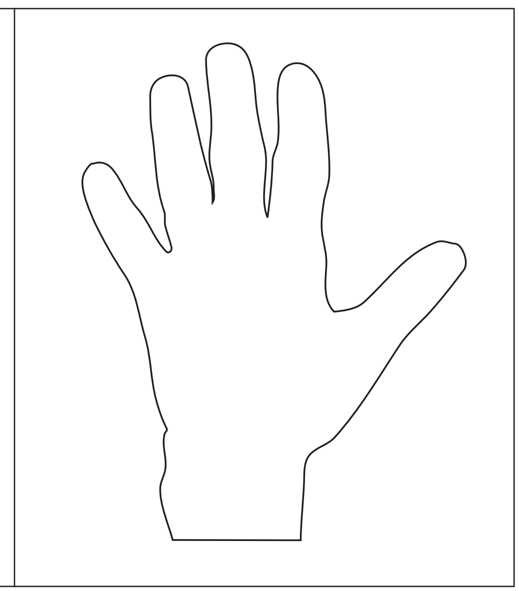




More



Fewer



Equal

More, Fewer, and Equal

Dear Parent/Guardian:

In math, we have been exploring the concepts of *more*, *fewer*, and *equal*. Here is an activity that will help your child compare numbers, using ideas about *more*, *fewer*, and *equal*. For this activity, you will need a bowl of small objects (e.g., beans, macaroni, buttons, small play blocks).

Take a handful of objects and place the objects in a pile between you and your child.

Challenge your child to create three other piles:

1. a pile that has fewer objects, to show the idea of *fewer;*
2. a pile that has a greater number of objects, to show the idea of *more;*
3. a pile that has the same number of objects, to show the idea of *equal*.

Watch how your child creates the three piles.

- Does your child count the objects in your pile first, and then create piles to show *fewer*, *more*, and *equal*?
- Does your child create piles with an appropriate number of objects?
- Can your child explain how his or her piles show ideas about *fewer*, *more*, and *equal*?

Switch roles. Have your child take a handful of objects and then watch you as you create three piles to show *fewer*, *more*, and *equal*. Have your child check the three piles to make sure that they contain an appropriate number of objects.

Paper Plate Dot Arrangements

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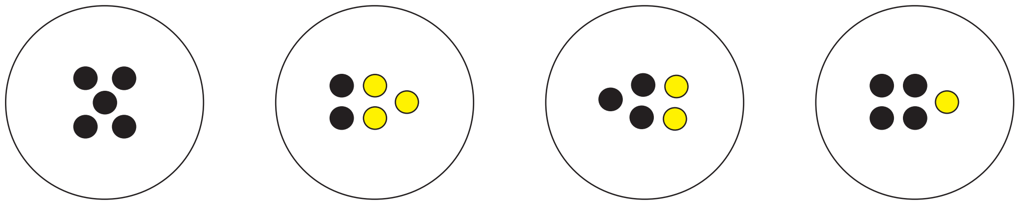

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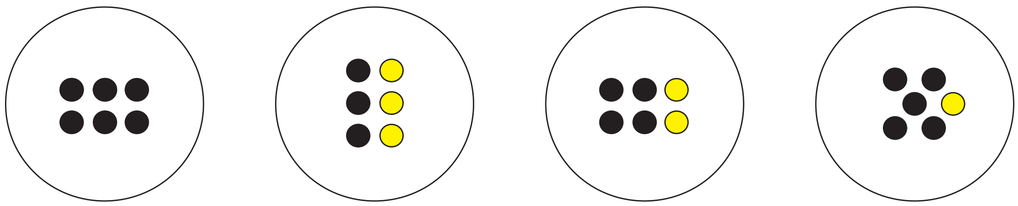

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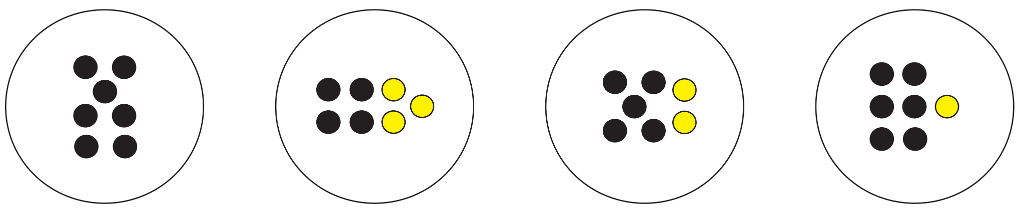

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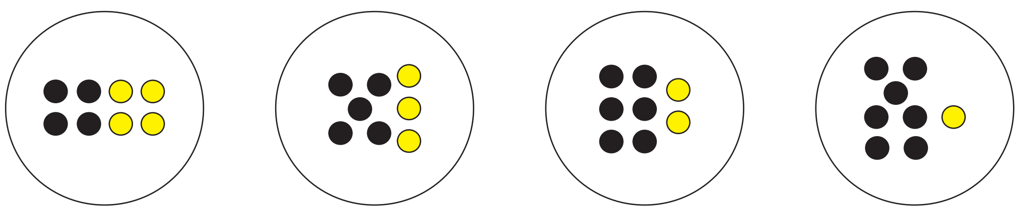

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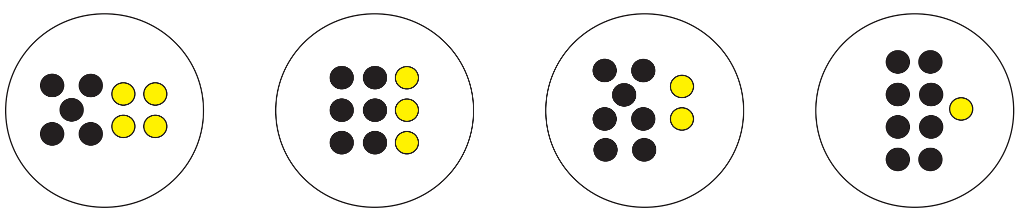

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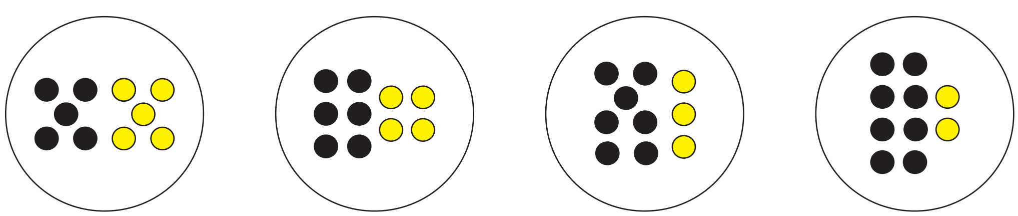
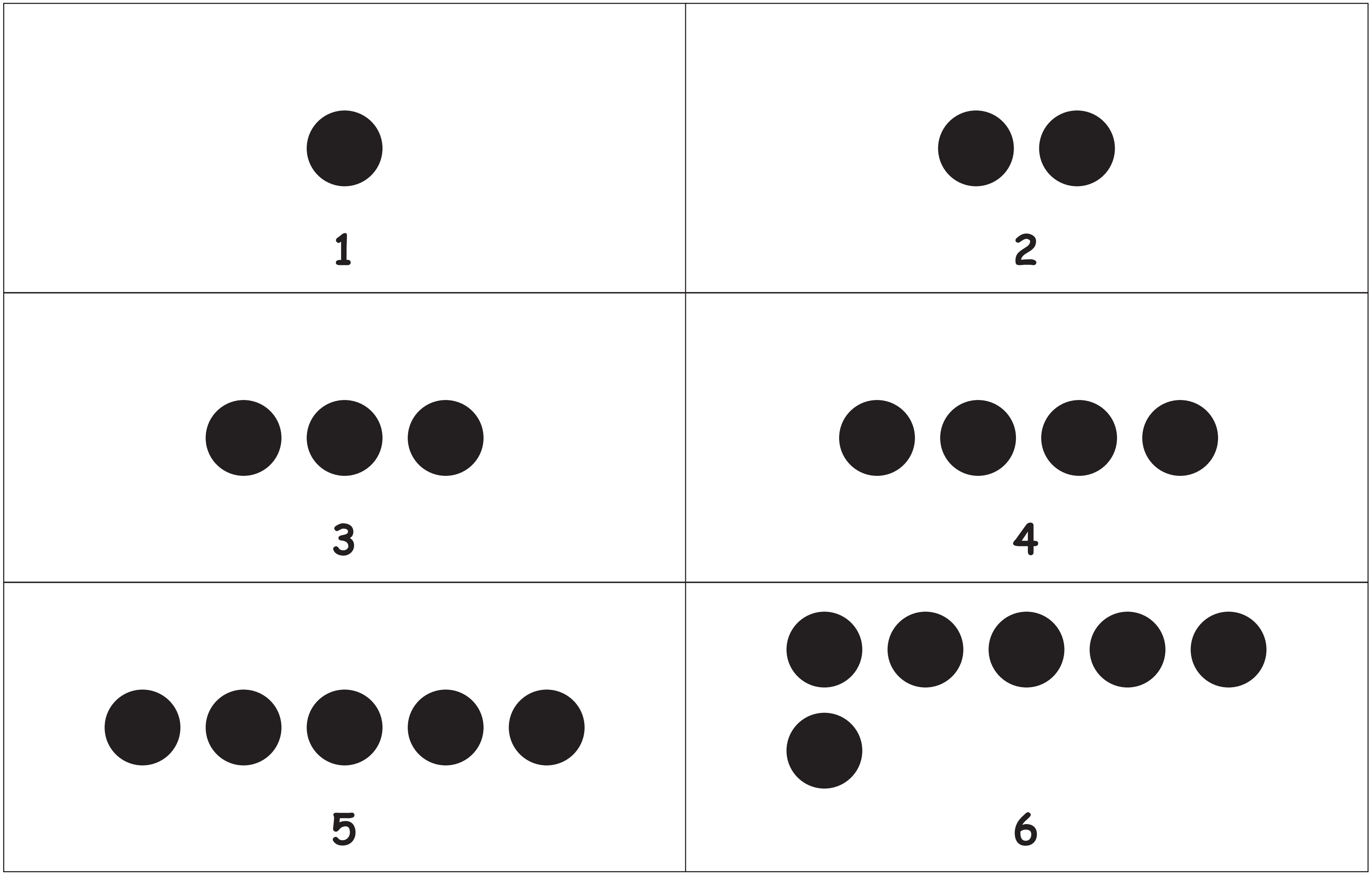
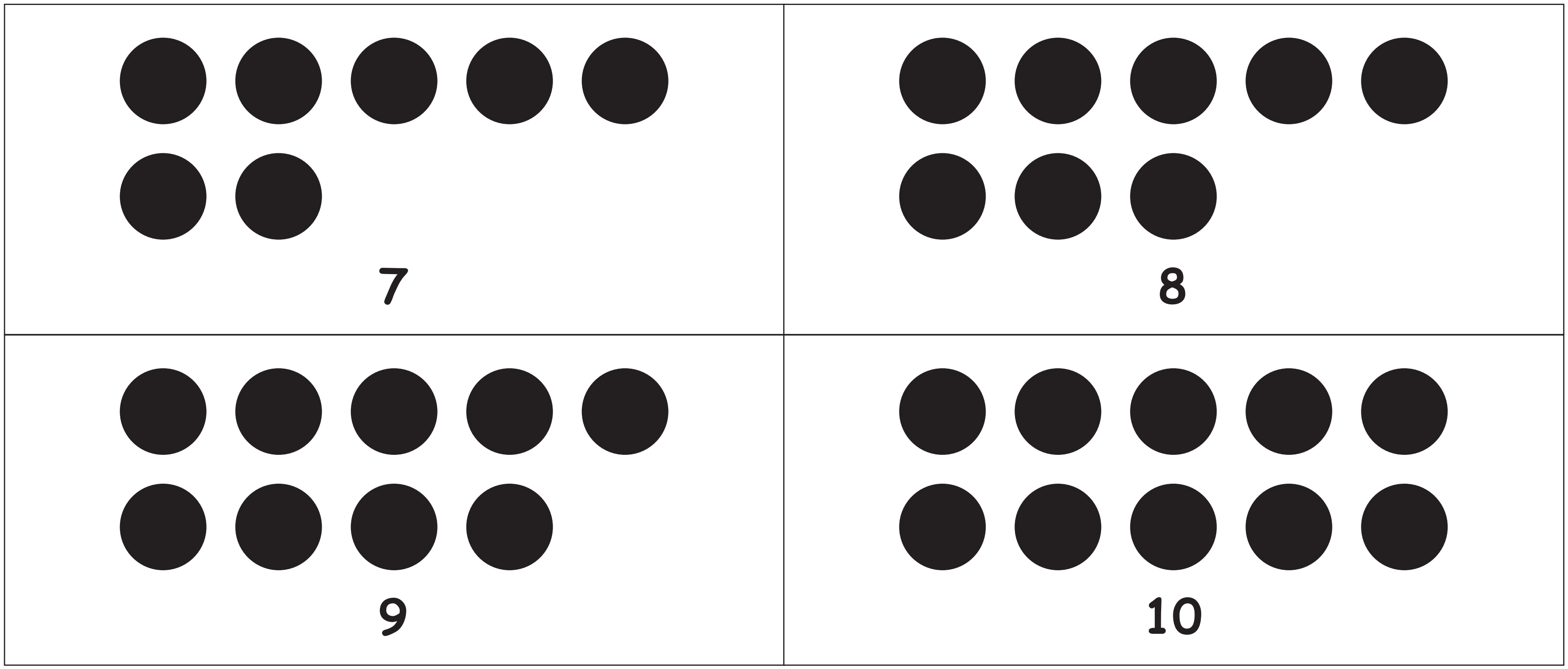


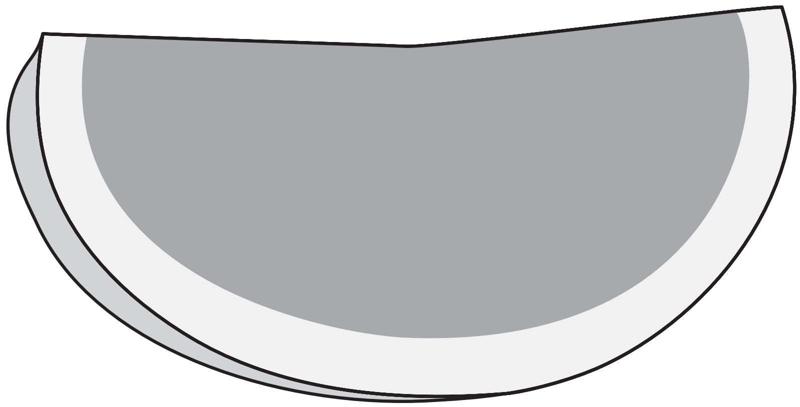
Number Dot Cards



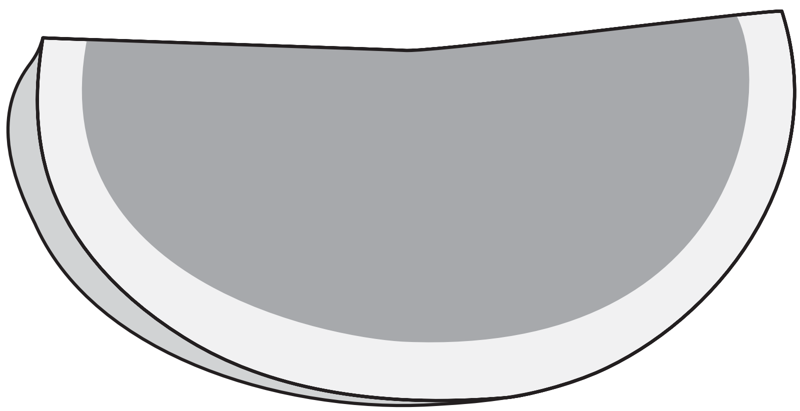
Number Dot Cards



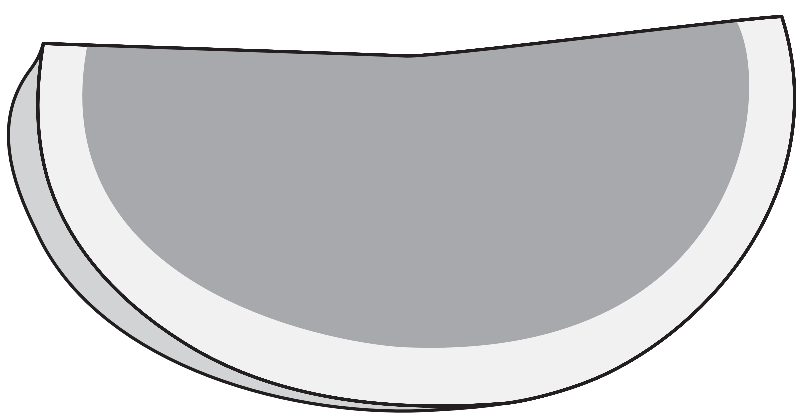
Watermelon Slices



More



Fewer



Equal

B. Grade 1 Learning Activities

Appendix Contents

Dragon Tail Patterns

BIG IDEA **Patterns and Relationships**

CURRICULUM EXPECTATIONS

Students will:

- identify, describe, and extend, through investigation, geometric repeating patterns involving one attribute (e.g., colour, size, shape, thickness, orientation);
- identify a rule for a repeating pattern (e.g., "We're lining up boy, girl, boy, girl, boy, girl.");
- create a repeating pattern involving one attribute (e.g., colour, size, shape, sound).

MATERIALS

- a large quantity of interlocking cubes of various colours
- PR1.BLM1: Grid Paper, cut vertically into long strips (2 strips per student)
- crayons
- white paper cut into cloud shapes approximately 8 cm long and 5 cm wide (1 paper cloud shape per pair of students)
- PR1.BLM2: What Part of the Pattern Is Covered?

ABOUT THE MATH

In the early primary grades, students develop an understanding of the repetitive nature of patterns as they identify, describe, and create geometric patterns (e.g., using concrete materials such as interlocking cubes, pattern blocks, and colour tiles). They learn to recognize the changing attribute(s) in patterns (e.g., colour, size, shape), and to identify the pattern core (i.e., the part of the pattern that repeats). They also learn to describe the rule for a pattern (e.g., describe the structure of a repetitive pattern). Understanding pattern structures allows students to extend patterns and to make conjectures about missing parts of a pattern.

In the following learning activity, students create colour patterns, using interlocking cubes, and then record their patterns by colouring squares on strips of paper. The activity also provides students with an opportunity to identify a missing part of a pattern.

GETTING STARTED

Gather students together in a discussion area in the classroom. Establish a context for the learning activity by telling the following story:

> "Thor is a dragon who lives in a town called Nogardville. Thor used to feel sad because the other dragons in Nogardville had beautiful colour patterns on their tails. But now, Thor is feeling happy because he finally has a chance to have his tail painted. But he can't decide which colours to choose or how to arrange the colours on his tail. Thor would like you to create some possible patterns for his tail."

Demonstrate how to use interlocking cubes to create a colour pattern for Thor's tail. Create a simple AB pattern (e.g., blue, green, blue, green, blue, green). After arranging six cubes in a row, ask: "What colour comes next on this dragon tail? How do you know?" Provide for individual students an opportunity to take turns adding an interlocking cube to the dragon tail. Ask the students to explain how they decided which colour to add to the tail.

Have the class read the pattern aloud (e.g., "blue, green, blue, green, blue, green, …"), pointing to each block as its colour is said. Ask:

- "Which part of the pattern keeps repeating?"
- "How many blue cubes are there in the part that repeats?"
- "How many green cubes are there in the part that repeats?"
- "What is the pattern rule?"

Show the students how to record the pattern by colouring squares on a strip of paper cut from PR1.BLM1: Grid Paper.

Repeat the activity by creating a more complex pattern for Thor's tail, using three colours (e.g., red, red, yellow, blue, red, red, yellow, blue, red, red, yellow, blue, …). Have students describe and extend the pattern.

Note: Present at least three complete repetitions of the pattern core to help students recognize the structure of the pattern.

Provide students with interlocking cubes, paper strips cut from PR1.BLM1: Grid Paper, and crayons. Challenge students to create a colour pattern for Thor's tail, using two or three colours of interlocking cubes. Allow them to decide on the length of their dragon tail. Instruct students to record their pattern by colouring strips of paper. (Students may tape strips together if they require a longer strip of paper.)

As students create and record their patterns, ask the following questions:

- "What is the pattern in your dragon tail?"
- "What is the pattern rule?"
- "What is the core of your pattern (the part of the pattern that repeats)?"
- "What comes next? How do you know?"
- "What other patterns can you make with two (three) colours of cubes?"

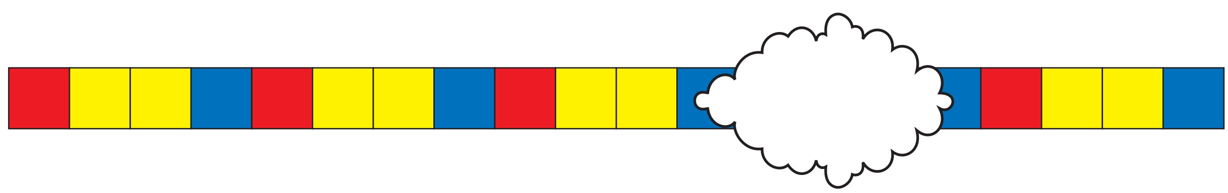
Provide assistance to students who have difficulty extending their patterns or recording their patterns on strips of paper.

WORKING ON IT

After students have created and recorded their patterns, explain the following:

> "I think Thor is going to like your colour tail patterns. He will likely be so happy that clouds of smoke will come out of his snout. That's what happens to dragons when they get really excited about something! Let's pretend that a cloud of smoke came out of Thor's snout and covered part of his tail."

Show students a colour pattern recorded on a strip of paper. Cover two or three elements of the pattern with a paper cloud of smoke.



Ask: "What part of the pattern is covered? How do you know?"

Discuss the different strategies students use to determine the covered elements of the pattern. Remove the paper cloud, and have students check whether they correctly identified the covered part of the pattern.

Repeat the activity by covering other sections of the pattern and having students identify the covered part.

Provide each pair of students with a paper cloud. Have students take turns covering part of their colour tail pattern (the coloured strips of paper created in Getting Started) with the paper cloud and asking their partner to identify the covered part of the pattern. Students then remove the paper cloud to reveal the covered part of the pattern.

As students work on the activity, ask: "What part of the pattern is covered by the cloud of smoke? How do you know?"

REFLECTING AND CONNECTING

Ask a student to show his or her coloured paper strip to the class. Have students read the pattern aloud, and ask them to explain the pattern rule (*e.g.*, "The pattern repeats 2 green, 1 blue, three times"). Ask if anyone has a pattern that is like the pattern being shown. Students may find that patterns are alike if they include the same colours, have the same length, or involve the same pattern structure (e.g., ABB). Ask: "How are the patterns alike? How are they different?"

Cover part of the student's coloured strip, and ask students to identify the covered part. Have students explain how they are able to determine what is covered.

Continue the discussion by asking other students to show different pattern strips to the class. Discuss the patterns, and have students identify covered parts of the patterns.

ADAPTATIONS/EXTENSIONS

In this activity, students decide how complex their own patterns will be. It may be necessary to encourage some students to create simple patterns, if you observe them developing patterns that are too difficult for them to extend. Other students may need to be challenged to create more complex patterns.

Recording the pattern by colouring squares on paper strips may be difficult for some students. Support these students by showing how one cube at a time can be removed from the row of cubes and its colour recorded on the strip of paper. Students might also align the row of cubes next to the paper strip, and colour the squares, using one-to-one correspondence.

Some students may need guidance in identifying the missing (covered) part of a pattern. Show these students how they can read aloud the first core of the pattern, in order to determine the pattern structure before trying to identify the missing part.

Some students may need a greater challenge. Pair these students with classmates who are able to create complex patterns that involve changes to attributes other than colour. For example, students might use pattern blocks to create a pattern that involves changing shapes. Provide pairs with two or three paper clouds. Have one student cover parts of the pattern, using paper clouds, and then have the partner identify the covered elements in the pattern.

MATH LANGUAGE

- pattern
- repeating pattern
- core
- pattern rule
- extend

ASSESSMENT

Observe students in order to assess how well they:

- create and describe colour patterns;
- extend patterns;
- identify the core of the pattern (i.e., the part of the pattern that repeats);
- describe the pattern rule;
- identify the missing (covered) part of a pattern;
- explain strategies for finding the missing part of a pattern;
- compare patterns, and explain how they are alike or different.

HOME CONNECTION

Send home PR1.BLM2: What Part of the Pattern Is Covered? This Home Connection activity provides parents and students with an opportunity to discuss repeating patterns and to identify missing parts of patterns.

LEARNING CONNECTION 1

Translating Patterns

Materials

– a variety of concrete materials (e.g., interlocking cubes, pattern blocks, colour tiles, attribute blocks, counters, beads)

Display a simple repeating pattern, using interlocking cubes (e.g., red, yellow, red, yellow, red, yellow). Have students describe the pattern, and discuss with them the pattern structure (e.g., AB, AB, AB).

Provide access to a variety of concrete materials (e.g., interlocking cubes, pattern blocks, colour tiles, attribute blocks, counters, beads). Challenge students to create a pattern that has the same structure as the one displayed and described, using one kind of concrete materials.

Ask a few students to display their patterns. Have them explain how their patterns match the pattern made with interlocking cubes. Ask: "How could we create a pattern with the same structure by using sounds? By using actions? By using symbols?" Have students explain possible sound patterns (e.g., meow, woof, meow, woof, meow, woof), action patterns (e.g., sit, stand, sit, stand, sit, stand), and symbol patterns (e.g., XO, XO, XO).

Repeat the activity by presenting more complex patterns (e.g., AAB, AAB, AAB; ABC, ABC, ABC; ABBA, ABBA, ABBA), using interlocking cubes. Have students translate each pattern, using concrete materials, sounds, actions, and symbols.

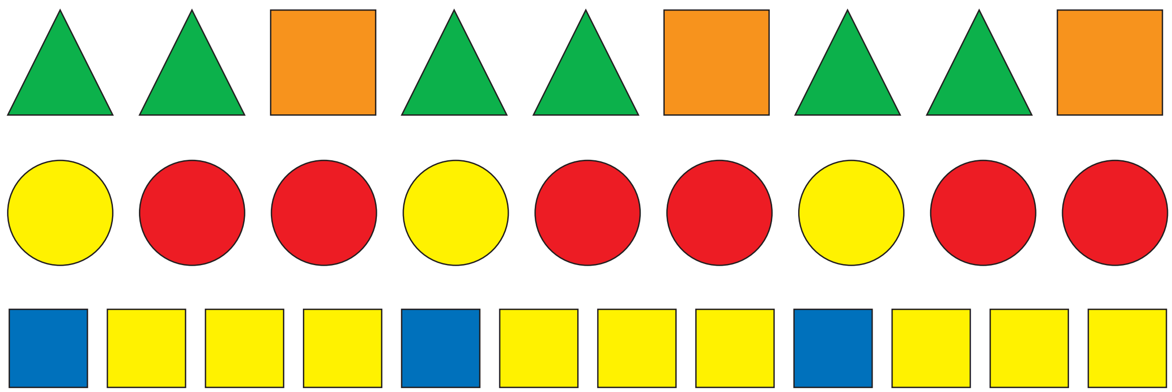
LEARNING CONNECTION 2

Which Pattern Is It?

Materials

- a variety of overhead materials (e.g., overhead pattern blocks, transparent counters, overhead colour tiles)
- overhead projector

On the board or the overhead projector, display three different patterns, using various concrete materials. Ensure that the patterns have different pattern structures.



Have half of the class close their eyes while the other half reads aloud a pattern that you point to (e.g., "ABB, ABB, ABB"). After hearing the pattern, students who closed their eyes may examine the patterns and identify the pattern that was read aloud.

LEARNING CONNECTION 3

Finding Patterns in a Hundreds Chart

Materials

- PR1.BLM3: Hundreds Chart
- overhead projector
- transparent counters

Display a hundreds chart on the overhead projector. Ask students to describe any patterns they observe (e.g., "All numbers in the fourth column have a 4 in the ones place"; "All numbers in the fourth row except the last number have a 3 in the tens place").

Next, have the students count aloud by 2's. Place a counter on each number as it is said aloud. Ask students to describe patterns that they observe (e.g., "The counters are arranged in vertical rows"; "All numbers with 2, 4, 6, 8, and 0 in the ones place are covered").

Next, have students close their eyes. Move a few counters from their spot on multiples of 2 onto an adjacent space in the hundreds chart. Have students examine the hundreds charts and identify which counters were moved.

Repeat the activity by having students count by 5's. Have students describe patterns created by counters that are placed on the multiples of 5. Again, move a few counters while students are not looking, and then ask them to tell which counters have been moved.

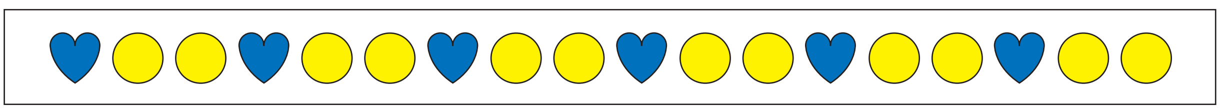
LEARNING CONNECTION 4

Patterned Headbands

Materials

- strips of paper approximately 10 cm x 50 cm, for headbands (1 per student)
- shapes (e.g., circles, squares, triangles, hearts, stars) cut from sponges
- trays of paint
- stapler

Show students how to create a patterned headband: select 2 or 3 sponge shapes, dip the shapes into trays of paint, and then stamp the shapes onto the headband strip. Stress the importance of identifying a pattern core and then repeating the core. Otherwise, students may choose shapes and colours randomly, without creating a pattern.



Give each student a blank headband, and have the students create their headband patterns. After the paint has dried, staple the two ends of each headband together.

Have students wear their headbands. Ask each student to find a classmate who has a headband pattern that is like his or her own in some way. For example, a student might find a classmate whose pattern involves the same colours, shapes, and/or pattern structure. Have pairs explain to the class how their headbands are alike.

Grid Paper

PR1.BLM1

What Part of the Pattern Is Covered?

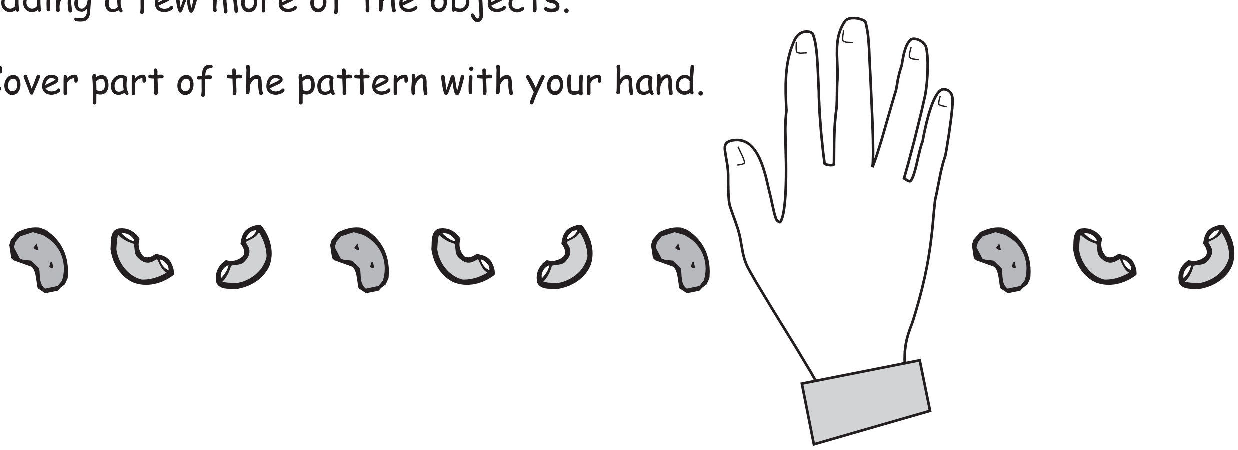
Dear Parent/Guardian:

We have been exploring patterns at school. Ask your child to explain how the class used coloured cubes to create patterns.

Ask your child to colour the squares in the following patterns. Ask him or her to describe each pattern, and to explain how the pattern could be extended.

Next, create a repeating pattern, using small objects such as beans and macaroni (e.g., bean, macaroni, macaroni, bean, macaroni, macaroni, bean, macaroni, macaroni). Have your child continue the pattern by adding a few more of the objects.

Cover part of the pattern with your hand.



Ask your child to identify the covered part of the pattern, and to explain how he or she knew what was covered.

Together, create other patterns, using small objects. Cover different parts of the pattern, and have your child explain what is covered.

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Balance It!

BIG IDEA Expressions and Equality

CURRICULUM EXPECTATIONS

Students will:

- determine, through investigation using a "balance" model and whole numbers to 10, the number of identical objects that must be added or subtracted to establish equality.

MATERIALS

- balances (1 per pair or small group of students)
- a large quantity of metal washers or any other small, heavy objects, such as marbles or pennies
- spinners made of EE1.BLM1: Balance It! Spinner, a paper clip, and a pencil (1 spinner per pair of students)
- EE1.BLM2: Balance It! Recording Sheet (1 per pair of students)
- EE1.BLM3: More, Less, Equal (1 per student)

ABOUT THE MATH

Equality is a complex concept for young students. To determine equality, students need to recognize whether one set of objects or a numeral represents a quantity that is the same as a quantity represented by another set of objects or another numeral. Often, young children are adept at comparing groups of objects, using concepts of *more* and *less*. However, they may have difficulties determining whether quantities are equal.

The balance provides an effective instructional tool for promoting students' understanding of equality. Experiences with the balance allow students to grasp concepts about equality by exploring situations of balance and imbalance, using concrete materials. An understanding of equality is essential if students are to develop more complex algebraic reasoning in later grades.

GETTING STARTED

Show students a balance and 5 washers. Count the washers aloud as you place them on the left pan. Ask students to explain why the left side of the balance dropped. Students might provide various explanations – for example, "This side is heavier than that side" or "This side is empty and the other one is not."

Ask: "What will happen if I put a washer on the right side of the balance?" Have students describe their predictions – for example, "The right side won't move" or "The right side will go down a bit." Place a washer on the right side of the balance, and have students explain whether their predictions were correct. Discuss how the sides are not balanced, because they do not contain an equal number of washers. Note: Washers must be of equal size and weight.

Continue to place 1 or 2 washers at a time on the right side of the balance. Before placing the washer(s), have students predict what will happen to the balance. Each time you add more washers, ask: "Are the sides balanced now? Why?/Why not?" Once both sides contain 5 washers, discuss how the sides are balanced because they contain an equal number of washers.

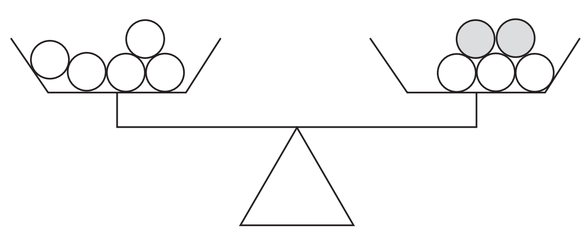
Provide pairs or small groups of students with a balance and washers. Have them experiment with finding ways to balance the balance, using different numbers of washers.

WORKING ON IT

Arrange students in pairs. Explain the activity:

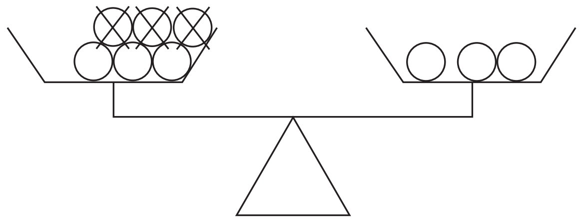
- One student spins the spinner (made with EE1.BLM1: Balance It! Spinner, a paper clip, and a pencil), and places on the left side of the balance the number of washers indicated by the spinner. On their individual copies of EE1.BLM2: Balance It! Recording Sheet, both students draw small circles to represent the washers on the left side of the balance.
- The other student spins the spinner and places on the right side of the balance the number of washers indicated by the spinner. On their recording sheets, both students draw the washers on the right side of the balance.
- The students determine whether the sides of the balance are balanced or not. In order to balance the sides, the students need to decide whether to add washers to one side or to remove washers from the other side.
- Students record their solutions on their recording sheets. Instruct students to draw and colour any washers they add, and to put an "X" on washers they remove. Have students explain in writing how they made the sides balance.

Example 1:



I added 2 washers.

Example 2:



I took away 3 washers.

Provide for students time to perform the activity several times. Provide extra copies of EE1.BLM2: Balance It! Recording Sheet if needed.

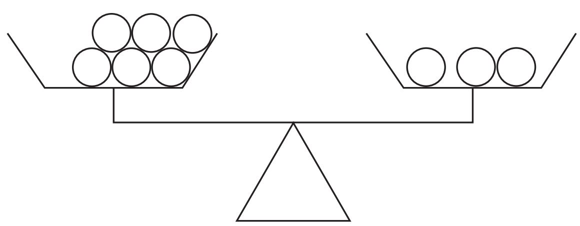
REFLECTING AND CONNECTING

Gather students together to discuss the activity. Ask questions such as the following:

- "When was the balance balanced?"
- "When was the balance not balanced?"
- "What did you need to do to make both sides balanced?"
- "How did you decide whether to add washers or take away washers?"

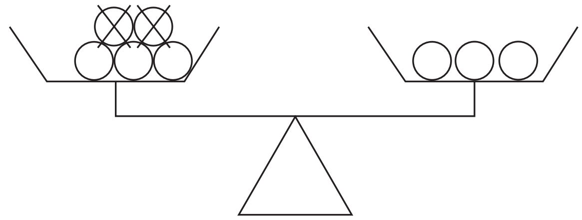
Have pairs of students show their work on EE1.BLM2: Balance It! Recording Sheet to the class. Ask pairs to choose one example and to explain how they made the sides balance.

On the board, draw a diagram of a balance and washers similar to the following:



Ask: "What could we do to make the sides balance?" For the example illustrated above, students might suggest adding 3 washers to the right side of the balance. Draw 3 washers on the right side of the balance, using a different colour. Explain that the left side contains 6 washers and the right side contains 3 washers plus 3 washers. Ask students how they might use numbers and symbols to record this idea. Discuss how $6 = 3 + 3$ represents the equal quantities on both sides of the balance.

Discuss other examples of equality, by relating equal quantities on the sides of a balance to an equation. The following example shows how the idea of taking away washers from one side to achieve balance can be expressed as an equation.



$5 - 2 = 3$

Some students may not fully understand ideas of equality expressed by symbolic equations. It is important to use simple language to explain the meaning of equations (e.g., "This side is the same amount as this other side"), and to relate concrete and pictorial representations to symbolic notations.

ADAPTATIONS/EXTENSIONS

It may be difficult for some students to understand how to balance the balance by using subtraction (i.e., by removing washers from a pan). Have these students use addition to achieve balanced sides (i.e., by adding washers to a side of the balance).

Challenge students to find two ways to balance the balance, using both addition and subtraction. For example, if the balance has 5 washers on the left side and 7 washers on the right side, students could add 2 washers to the left side or subtract 2 washers from the right side. Have students record the corresponding equations for both operations (e.g., $5 + 2 = 7$, $5 = 7 - 2$).

MATH LANGUAGE

- balance
- equal
- equation
- add, addition
- subtract, subtraction

ASSESSMENT

Observe students to assess how well they:

- find ways to balance the balance (e.g., by adding or removing washers);
- explain concepts about equality (e.g., "The same quantity is on both sides");
- use appropriate mathematical language to describe equality and inequality (e.g., *equal*, *same*, *less than*, *greater than*, *more*, *fewer*);
- explain equality in addition and subtraction sentences (e.g., $2 + 3 = 5$ means that 2 plus 3 is the same quantity as 5).

HOME CONNECTION

Send home EE1.BLM3: More, Less, Equal. In this Home Connection activity, parents and students compare sets of objects, using *more*, *less*, and *equal*. The activity also allows students to determine how to create equal sets by adding objects to (or removing objects from) one set.

LEARNING CONNECTION 1

Matching Cards

Materials

- EE1.BLM4: Number Cards (1 set per pair of students)
- EE1.BLM5: Dot Cards (1 set per pair of students)

Provide each pair of students with a set of number cards made from EE1.BLM4: Number Cards and a set of dot cards made from EE1.BLM5: Dot Cards. Explain the game:

- Students place each set of cards in a separate pile face down between them.
- Players take turns flipping the top card in both piles. If the number card and the dot card show equal quantities, the player gets to keep the cards. If the cards do not show equal quantities, the player reinserts each card into the appropriate pile.
- The game continues until all the cards have been claimed by the players.
- The player with the most cards wins the game.

As students play the game, ask them to explain how they know which number cards and dot cards go together. Emphasize the idea that matching number and dot cards represent equal quantities.

LEARNING CONNECTION 2

Creating Equal Sets

Materials

- small containers holding at least 20 counters (1 container per pair of students)
- sheets of paper (1 per pair of students)

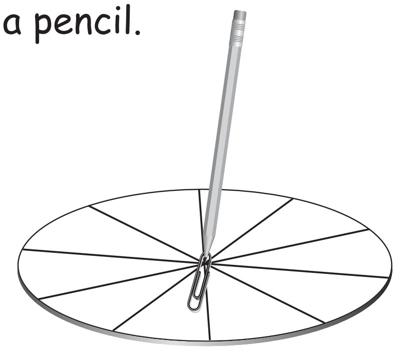
Arrange students in pairs. Provide each pair with a small container of counters. Player A secretly takes 1 to 10 counters and places them under a sheet of paper while player B looks away. Player B then guesses the number of counters under the paper and places the same number of counters beside the paper. The players lift the sheet of paper and compare the number of counters in both sets. If the sets of counters are equal, player B earns a point. If the sets are not equal, the players place the sheet of paper back onto player A's counters, and player B adjusts the number of counters in his or her set by either adding or subtracting counters. The players remove the sheet of paper again. If both sets are equal, player B then earns a point. If the sets are not equal, player B earns no points.

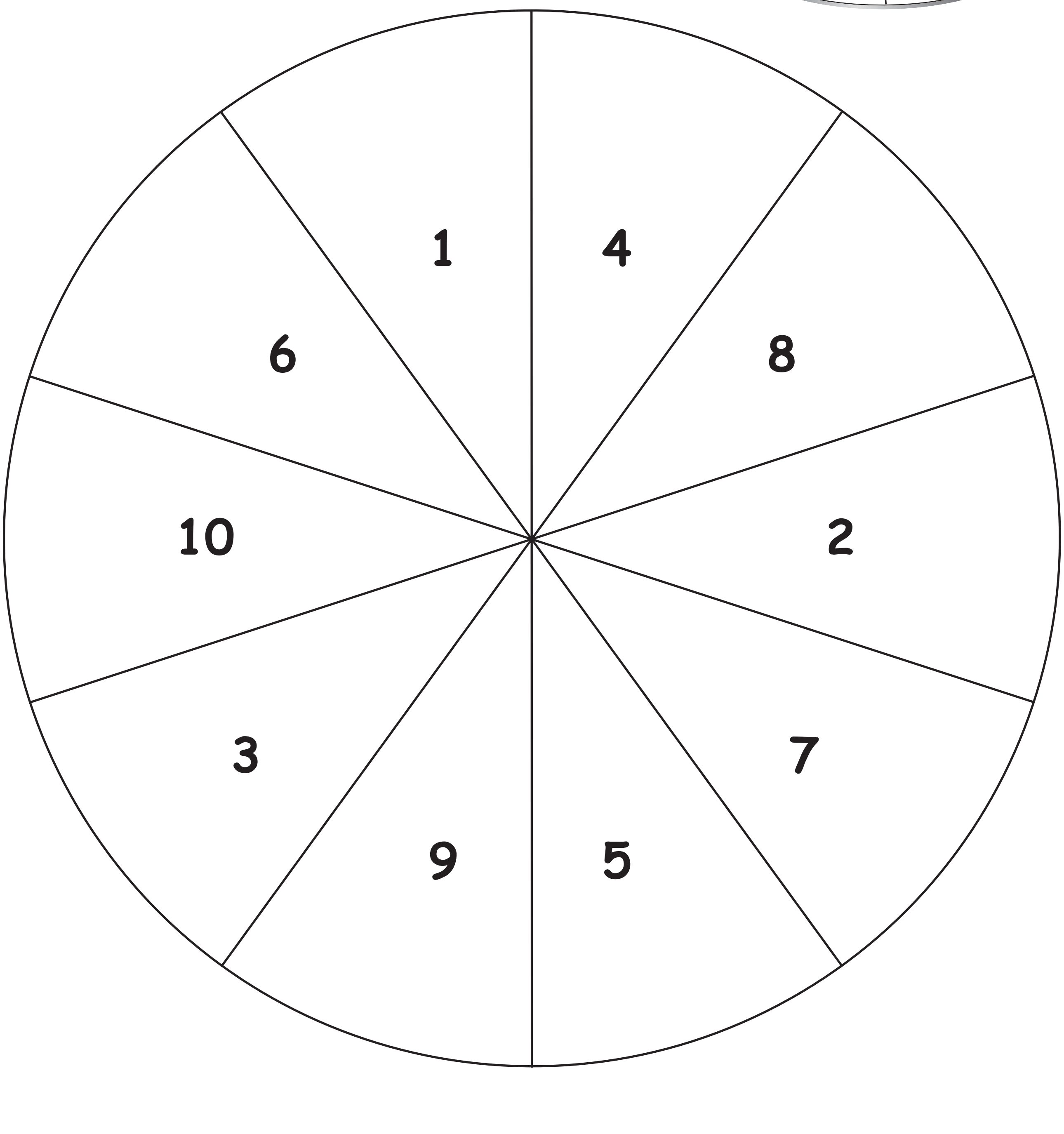
Students continue to switch roles as they play the game. The first player to earn a total of 10 points wins the game.

As students play the game, observe how well they are able to adjust the number of counters (by adding and subtracting counters) to make equal sets.

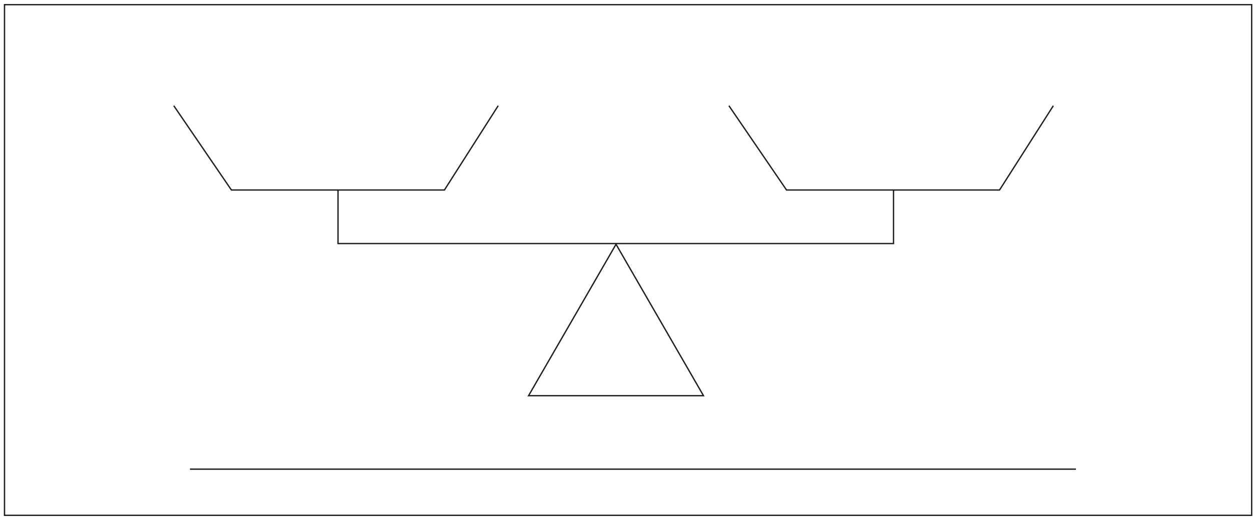
Balance It! Spinner

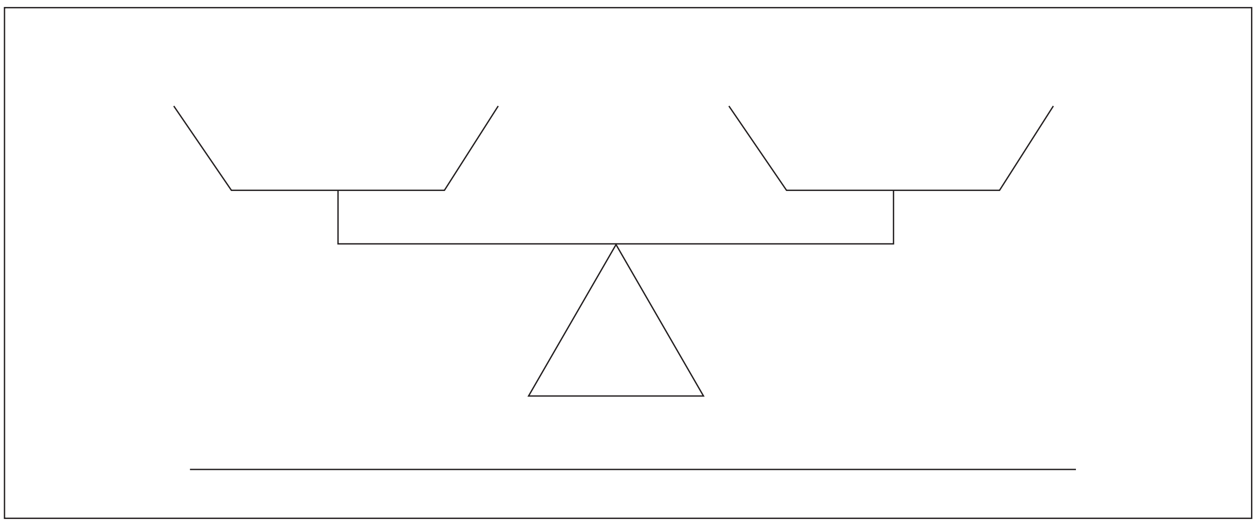
Make a spinner, using this page, a paper clip, and a pencil.

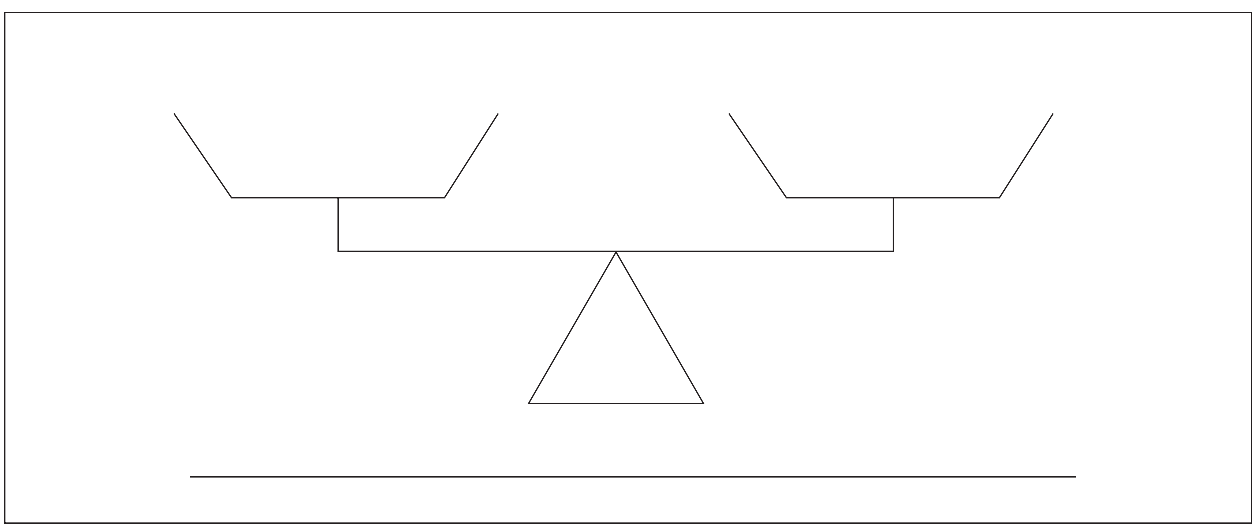




Balance It! Recording Sheet







More, Less, Equal

Dear Parent/Guardian:

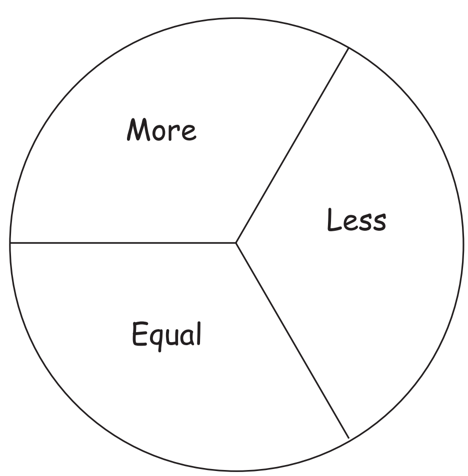
We have been learning how to use the words *more*, *less*, and *equal* to compare numbers (e.g., "5 is more than 3", "2 is less than 7", "4 is equal to 4").

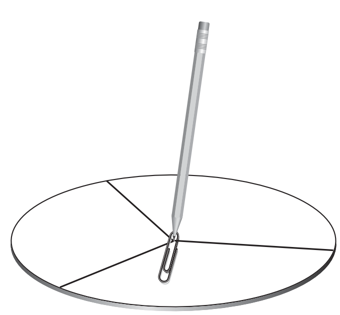
Play the following game with your child to help him or her understand *more*, *less*, and *equal*. For this game, you will need:

- a bowl of small objects (e.g., beans, macaroni, buttons, small play blocks);
- a spinner made with a paper clip, a pencil, and the spinner circle below;
- paper and pencil to keep track of the points.

How to Play the Game

- Each player takes a handful of the small objects and places the objects in a pile in front of him or her.
- One player spins the "More/Less/Equal" spinner.
- If the spinner lands on "More", the player who has more objects in his or her pile scores a point.
- If the spinner lands on "Less", the player who has fewer objects in his or her pile scores a point.
- If the spinner lands on "Equal", the players work together to make their handfuls equal by adding more objects to one of the piles (or taking away some objects from one of the piles). If the players successfully create equal piles, both players receive a point.
- Players return their handful of items to the bowl and play again.
- The game continues until one player has earned 10 points.

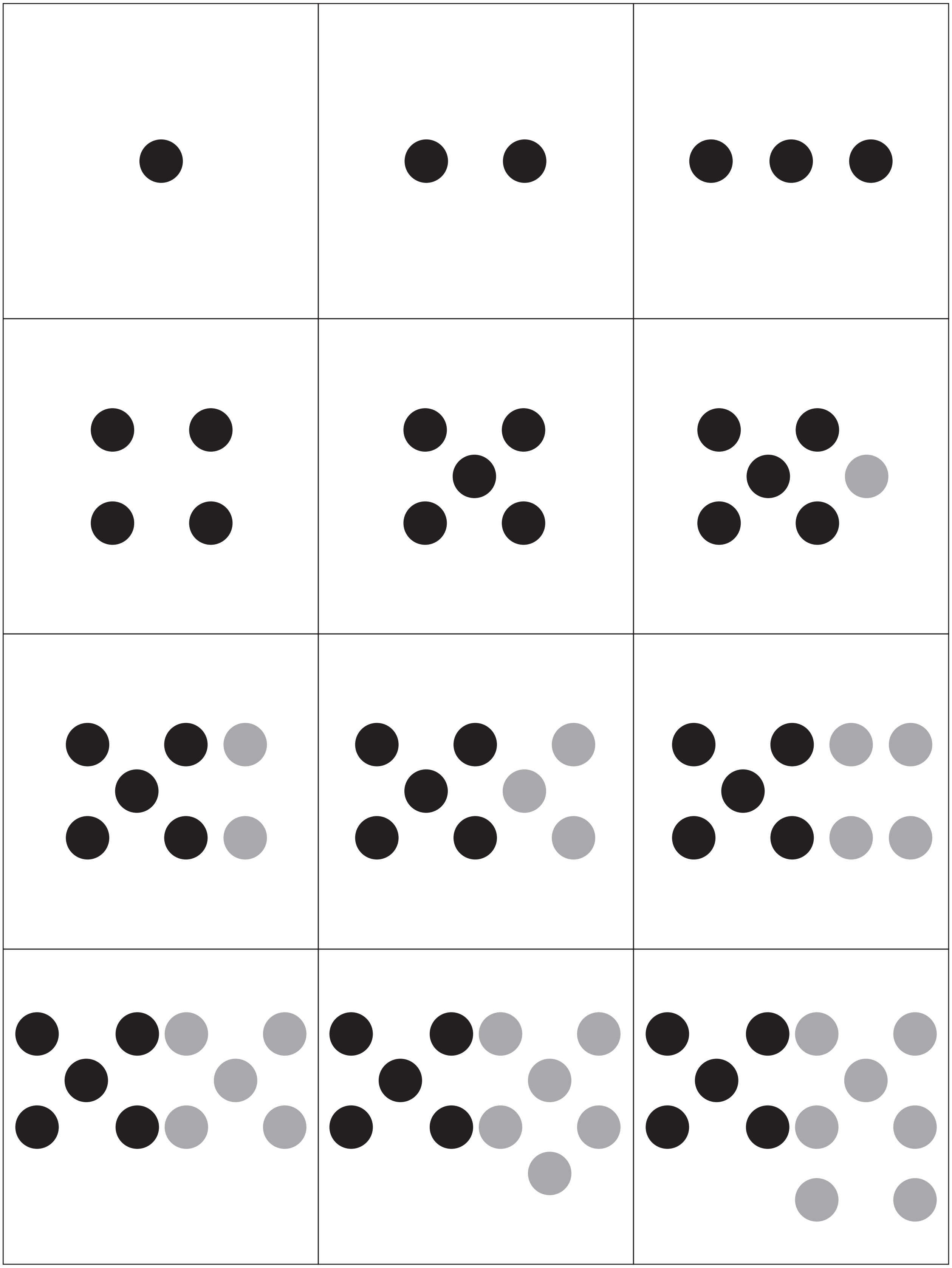




Number Cards

1	2	3
4	5	6
7	8	9
10	11	12

Dot Cards



C. Grade 2 Learning Activities

Appendix Contents

Growing T's

BIG IDEA Patterns and Relationships

CURRICULUM EXPECTATIONS

Students will:

- identify, describe, and create, through investigation, growing patterns and shrinking patterns involving addition and subtraction, with and without the use of a calculator (e.g., $3+1=4$, $3+2=5$, $3+3=6$, …);
- identify repeating, growing, and shrinking patterns found in real-life contexts (e.g., a geometric pattern on wallpaper, a rhythm pattern in music, a number pattern when counting dimes);
- represent a given growing or shrinking pattern in a variety of ways (e.g., using pictures, actions, colours, sounds, numbers, letters, number lines, bar graphs);
- create growing or shrinking patterns.

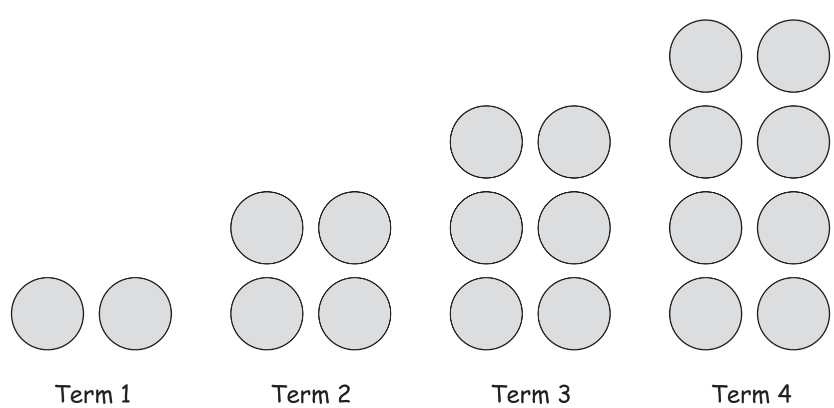
MATERIALS

- a large quantity of coloured counters
- PR2.BLM1: Growing T's (1 per student)

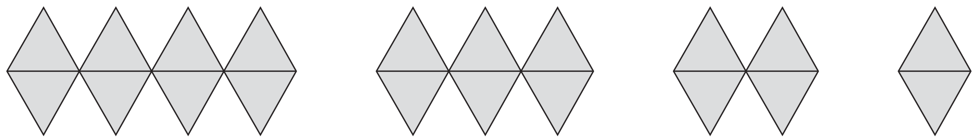
ABOUT THE MATH

In Grade 2, students identify, describe, and create repeating patterns. They identify the core of a pattern (i.e., the part of the pattern that repeats), and use this information to extend patterns and to predict what comes next in a pattern. They represent patterns in various ways (e.g., using pictures, actions, colours, letters), and represent the same pattern structure in different forms.

In Grade 2, students also begin to explore growing and shrinking patterns. In a growing pattern, the number of elements increases from term to term.



In shrinking patterns, the number of elements decreases from term to term.



Opportunities to create, describe, and extend growing and shrinking patterns help students relate the increase or decrease of elements in a pattern to addition (e.g., "Each time, you add 2 more circles") and to subtraction (e.g., "Each time, you subtract 2 triangles").

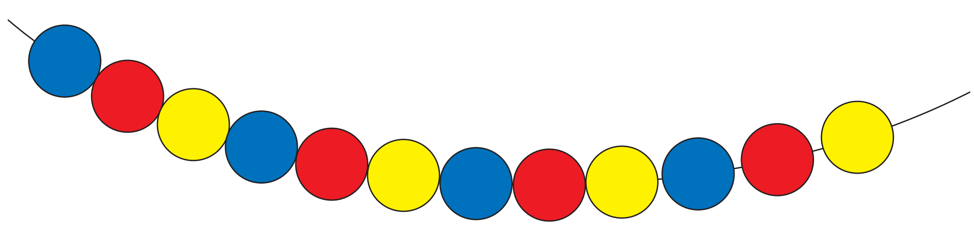
In the following learning activity, students create repeating patterns and extend a growing pattern. The activity provides students with an opportunity to recognize that a repeating pattern involves the repetition of a pattern core, whereas a growing pattern involves an increase in the number of elements as the pattern progresses. By discussing how a pattern grows, students are able to describe how the pattern is related to addition (e.g., "You add 1 more bead to the pattern every time").

GETTING STARTED

Establish a context for the learning activity by explaining the following:

> "Tina and her T-ball team were given a set of water bottles. The water bottles were identical, and during a game, players often got confused about which water bottle was their own. Tina had an idea: 'Why don't we each create a pattern on our own water bottle. We could arrange beads on a string in a pattern, and then tie the strings to the bottles. That way, we can tell which water bottle is our own by looking at the bead pattern.'"

On the board or chart paper, draw a line to represent a string. Draw coloured beads in a repeating pattern (e.g., blue, red, yellow, blue, red, yellow, blue, red, yellow). After drawing a few beads, invite students to identify the pattern and to suggest the next few beads in the pattern sequence.



Provide students with containers of coloured counters, and explain that the counters represent beads. Invite students to use the counters to create a pattern that Tina or her teammates might have created.

Have students show and explain their patterns to a partner. Guide the discussion between partners by asking the questions listed below. Pose one question at a time, and provide time for partners to discuss their answers. Use a signal, such as a hand clap, after each discussion time, to gain students' attention before posing the next question.

- "What is the core (i.e., the part of the pattern that repeats) of your pattern and of your partner's pattern?"
- "How many times does the core repeat in your pattern?"
- "How are your pattern and your partner's pattern alike?"
- "How are your pattern and your partner's pattern different?"

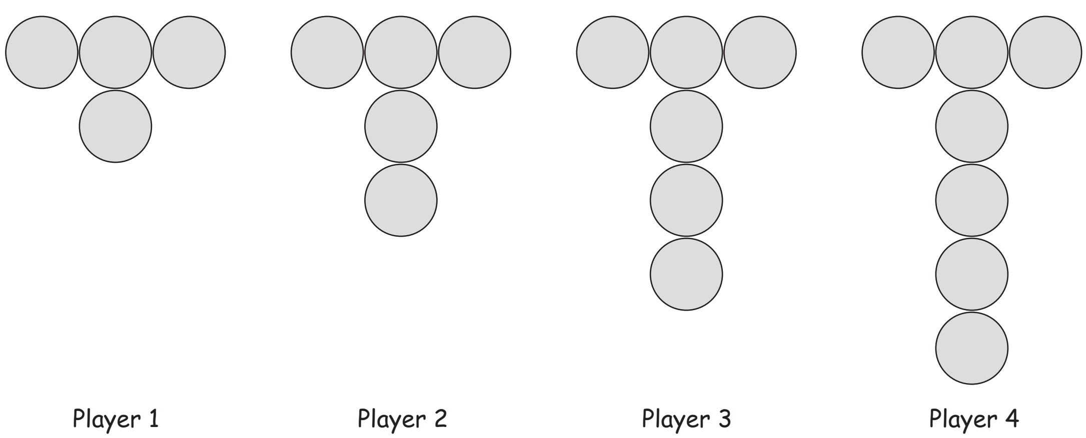
Ask the class: "Do you think that Tina's idea will allow team members to tell their water bottles apart?"

WORKING ON IT

Continue to explain the situation about Tina and her teammates:

> "Tina and her teammates created the bead patterns and attached the strings of beads to their water bottles. However, at the next game, players continued to mix up their water bottles, because so many of the patterns looked the same. Tina had another idea. "Why don't we use beads to create a different kind of pattern? Because we are a T-ball team, we could follow a pattern to create T's of different sizes. Then we will each have a different-sized T for our water bottles."

On the board or chart paper, illustrate a growing pattern. Explain how the beads, represented by circles, might be arranged to create different-sized T's for four players.



Discuss the growing pattern by asking the following questions:

- "What part of the T stays the same?"
- "What part of the T grows?"
- "What is the pattern?"

Invite students to use coloured counters to create the T's for the first four players. Explain that they can arrange the colours of the beads (counters) in any order. After students have created four T's, ask: "What would the T for player 5 look like? How do you know?" Have students create the T for player 5.

Provide students with a copy of PR2.BLM1: Growing T's. Instruct students to extend the growing pattern, using counters, and then to draw a picture of the T for each of players 4, 5, and 6. Ask them to record the number of beads that are needed to create each of the first six T's in the pattern. After students have completed the first six T's, have them find the number of beads that are needed for both the seventh and the eighth T. Direct students to explain on their worksheets how they found the number of beads.

As students work on the activity, ask them the following questions:

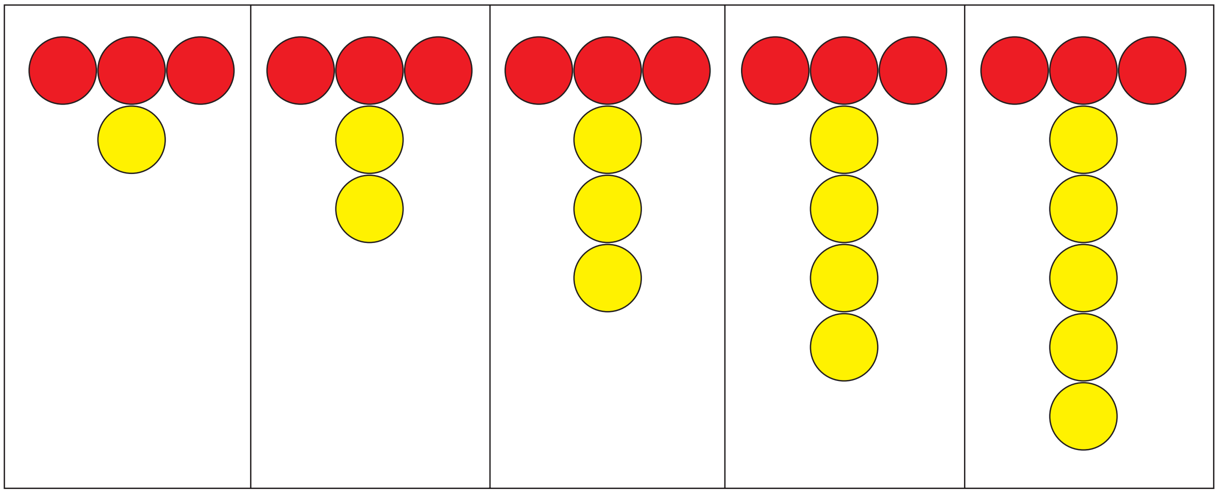
- "What is the pattern?"
- "Why is this pattern a *growing* pattern?"
- "How many beads do you think will be needed for the seventh T? Why do you think that?"
- "How many beads do you think will be needed for the eighth T? Why do you think that?"

Students may use a variety of strategies to find the number of beads that are needed for the seventh and the eighth T's. Some students may need to use counters to construct the T's. Other students may recognize and extend a numeric pattern.

REFLECTING AND CONNECTING

Gather students together to share their work. Ask a few students to explain how they were able to find the number of beads for the seventh and eighth T's in the pattern. Try to include students who used various strategies – for example, those who used counters to construct the T's and then counted the number of counters, and those who recognized and extended a numeric pattern.

Display the following chart on chart paper or on the board:



Relate the growing pattern in the chart to addition by asking questions such as the following:

- "How many red beads are there in the first T?"
- "How many yellow beads are there in the first T?"
- "How many beads are there altogether in the first T?"
- "How could we represent the number of red and yellow beads in the first T as an addition sentence?"

Record "3 + 1 = 4" below the first T. Continue to ask questions about the total number of beads in the other T's in the chart. Elicit from students the addition sentence for each T, and record it in the chart. Ask:

- "What part of the addition sentence stays the same?"
- "What part of the addition sentence changes?"
- "How does the addition sentence change?"

Discuss how one more bead is added to each new T. As a result, the number added to 3 increases by 1 each time (3 + 1, 3 + 2, 3 + 3, …), as does the sum (4, 5, 6, …).

ADAPTATIONS/EXTENSIONS

In the Getting Started part of the learning activity, students decide how complex their own patterns will be. It may be necessary to encourage some students to create simpler patterns, if you observe them developing patterns that are too difficult for them to extend. Other students may need to be challenged to create more complex patterns.

Encourage students who have difficulty extending the growing pattern to use counters to represent the pattern concretely. Help them understand how the pattern progresses by demonstrating that a counter is added at each new term. Some students may need to extend the pattern by using concrete materials, without recording the pattern on paper.

Challenge students to create other growing or shrinking patterns. First, have them use manipulatives to create patterns, and then ask them to draw a diagram of the pattern. Have students explain how the increase or decrease of elements in a pattern is related to addition or subtraction.

MATH LANGUAGE

- repeating pattern
- growing pattern
- shrinking pattern
- increasing
- decreasing
- pattern rule
- term

ASSESSMENT

Observe students to assess how well they:

- create, describe, and compare repeating growing and shrinking patterns;
- extend and describe growing and shrinking patterns;
- explain pattern rules;
- predict what comes next in a pattern;
- relate the increase of elements in a growing pattern to addition.
- relate the decrease of elements in a shrinking pattern to subtraction.

HOME CONNECTION

Send home PR2.BLM2: Growing Patterns. This letter encourages parents to explore growing patterns with their children.

LEARNING CONNECTION 1

How Many Feet?

Materials

- *How Many Feet in the Bed?* by Diane Johnston Hamm (New York: Simon & Schuster, 1991)
- sheets of paper (1 per pair of students)
- pencils (1 per pair of students)

Read *How Many Feet in the Bed?* to the class, if the book is available. As you read the story, have students identify the number of feet that are in the bed at different points in the story.

Relate the increasing number of feet in the bed to a growing pattern by asking questions such as the following:

- "How many people were in the bed at the beginning of the story?"
- "How many feet were in the bed then?"
- "How many people were in the bed when the little girl somersaulted into bed?"
- "How many feet were in the bed then?"

Together, create a chart on the board or on chart paper to show the growing pattern.

Number of People	Number of Feet
1	2
2	4
3	6
4	8
5	10

Ask students to describe patterns they observe in the chart. For example, students might notice that the growing pattern for the number of feet involves adding 2 for each new person.

Provide pairs of students with a pencil and a sheet of paper. Have them create a chart that shows the number of toes in the bed.

Number of People	Number of Toes
1	10
2	20
3	30
4	40
5	50

Discuss patterns in the chart.

LEARNING CONNECTION 2

Can You Create My Pattern?

Materials

– barrier (e.g., book, binder)
– a variety of manipulatives (e.g., counters, colour tiles, interlocking cubes)

Have pairs of students sit facing each other. Direct them to set up a barrier between them in order to block their work from each other's view. To begin, a student creates a growing pattern, using manipulatives. After creating 3 or 4 terms of the pattern, the student provides oral instructions to his or her partner on how to create an identical pattern without looking at the original pattern. Students might give instructions such as the following:

- "For the first term, place two red tiles beside each other."
- "For the second term, place two red tiles beside each other, and place a blue tile on top of them."
- "For the third term, place two red tiles beside each other, and add 2 blue tiles on top of them."

After all instructions have been given, students remove the barrier and compare their patterns. If the patterns are not identical, encourage students to discuss how the instructions might have been more precise.

LEARNING CONNECTION 3

Patterns, Patterns!

Materials

– overhead manipulatives (e.g., counters, colour tiles, pattern blocks)
– overhead projector
– a variety of manipulatives (e.g., counters, colour tiles, interlocking cubes, pattern blocks)

Ask a student to create a repeating pattern on the overhead projector, using overhead manipulatives. Have students describe the pattern and identify the pattern core (i.e., the part of the pattern that repeats). Discuss how they might read the pattern, using letters (e.g., ABB, ABB, ABB, …), and read the pattern together with the students.

Provide students with a variety of manipulatives (e.g., counters, colour tiles, interlocking cubes, pattern blocks). Instruct students to select one kind of manipulative and to create a pattern that has the same structure (i.e., follows the same letter sequence) as the pattern on the overhead projector. Invite a few students to display their patterns and to explain how they matched the pattern on the overhead projector.

Next, invite students to use a different manipulative to create a pattern that has the same structure. Again, discuss how students' patterns all follow the same letter sequence (e.g., all patterns follow an AAB, AAB, AAB sequence).

Provide students with opportunities to create a variety of patterns that involve the same structure. Have them use different manipulatives to represent the patterns. Discuss how the different patterns are based on the same pattern structure.

LEARNING CONNECTION 4

Predicting Numbers

Materials

- PR2.BLM3: Hundreds Chart (1 per pair of students)
- counters (several per pair of students)
- sheets of paper (1 per student)
- pencils (1 per student)

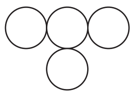
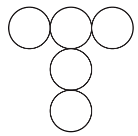
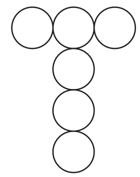
Explain the game to the class:

- Students play the game in pairs. Each pair of students needs a copy of PR2.BLM3: Hundreds Chart, and several counters.
- One student chooses a number between 1 and 10 and places a counter on the number.
- Next, both students count aloud by 2's beginning at the number chosen by the first student, and they place a counter on each number in the count. Students count as far as the number closest to 50, but no further than 50.
- Each student then predicts four numbers that will be covered by counters when the count continues beyond 50. Each records his or her four numbers on a piece of paper, without telling or showing the numbers to the partner.
- Players continue to count by 2's from the last covered number on the hundreds chart, placing counters on the numbers in the count.
- Students receive a point for every number that they predicted correctly.
- The game continues until one or both players score 20 points.

Variations of the Game

- Students may count by 5's or by 10's.
- Students may begin with a number between 90 and 100, and then count backwards by 2's, 5's, or 10's.

Growing T's

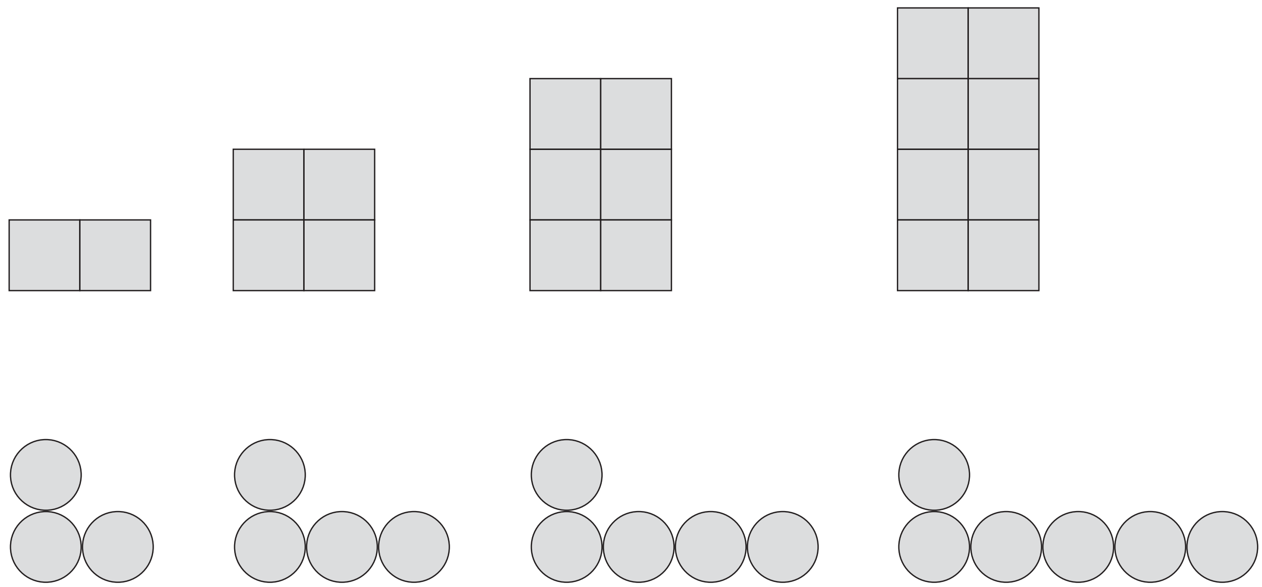
Player 1	Player 2	Player 3	Player 4	Player 5	Player 6
					
How many beads? ________	How many beads? ________	How many beads? ________	How many beads? ________	How many beads? ________	How many beads? ________

How many beads are needed for Player 7? ______ How do you know?	How many beads are needed for Player 8? ______ How do you know?

Growing Patterns

Dear Parent/Guardian:

We have been exploring growing patterns. Here are examples of growing patterns:



Create growing patterns with your child, using small items that you have at home (e.g., pennies, buttons, beans, macaroni). Ask your child to explain how the pattern grows. For example, he or she might explain, "You add 2 more pennies each time" or "The pattern is adding 1 each time."

You might also use small items to create the first four terms of a growing pattern and then ask your child to extend the pattern by constructing the next three terms. Ask your child to explain how he or she was able to extend the pattern.

Enjoy exploring growing patterns!

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Combo Quest

BIG IDEA **Expressions and Equality**

CURRICULUM EXPECTATIONS

Students will:

- demonstrate an understanding of the concept of equality by partitioning whole numbers to 18 in a variety of ways, using concrete materials (e.g., starting with 9 tiles and adding 6 more tiles gives the same result as starting with 10 tiles and adding 5 more tiles);
- represent, through investigation with concrete materials and pictures, two number expressions that are equal, using the equal sign (e.g., "I can break a train of 10 cubes into 4 cubes and 6 cubes. I can also break 10 cubes into 7 cubes and 3 cubes. This means 4 + 6 = 7 + 3.").

MATERIALS

- Cuisenaire rods (1 set per pair of students)
- spinners made of EE2.BLM1: Combo Quest Spinner, a paper clip, and a pencil (1 spinner per pair of students)
- sheets of paper (1 per pair of students)
- math journals
- EE2.BLM2: Equations Game (1 per student)

Note: If Cuisenaire rods are unavailable, create strips by using coloured paper and EE2.BLM3: Cuisenaire Rod Relationships.

ABOUT THE MATH

Since young students often view equations in a format such as 4 + 5 = 9, they can develop misconceptions about the meaning of the equal sign. Specifically, students may think that the equal sign means "gives an answer of", rather than "is the same quantity as". Given 4 + 5 = ___, students might answer "9", not because they consider the equality on both sides of the equation, but because they believe the equal sign is a prompt for an answer. Students' misconceptions about the equal sign are often not apparent to teachers until students are asked to complete an equation such as 3 + 5 = ___ + 2. Students may believe that "8" is the missing numeral in the equation because they think that the equal sign is asking them to add 3 and 5. Students who have misconceptions about the equal sign also have difficulty recognizing that equations in unfamiliar formats are true (e.g., 7 = 3 + 4, 5 + 2 = 1 + 6).

It is important that young students understand the following: (a) the equal sign is a symbol that separates both sides of an equation, and (b) the equal sign states that the right side of the equation is *the same quantity as* the left side (e.g., in 5 = 3 + 2, 5 is the same quantity as 3 + 2). When students understand that the equal sign describes the relationship of equality between two quantities, they begin to focus their reasoning on the quantities represented by both sides of the equation.

In the following learning activity, students investigate equalities, using Cuisenaire rods. After finding combinations of rods that represent the same value, they translate these ideas of equality into symbolic equations. The activity provides students with an opportunity to develop an understanding that the equal sign means "the same quantity as" in equations.

GETTING STARTED

Display a chart that shows the numeric values for the different colours of Cuisenaire rods.

white =	1
red =	2
light green =	3
purple =	4
yellow =	5
dark green =	6
black =	7
brown =	8
blue =	9
orange =	10

Arrange students in pairs. Provide each pair with a set of Cuisenaire rods and a spinner (made of EE2.BLM1: Combo Quest Spinner, a paper clip, and a pencil). Explain the activity:

- Students spin the spinner and select the Cuisenaire rod indicated on the spinner.
- Students find as many different combinations as possible of Cuisenaire rods that have the same length as the rod indicated by the spinner. For example, if the spinner shows "black", students might find, among others, the following combinations of Cuisenaire rods:

black		
red	yellow	
purple	light green	
red	red	light green

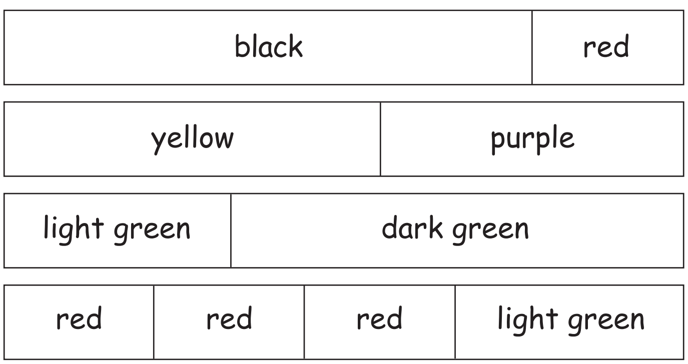
- On a sheet of paper, students record the different combinations of Cuisenaire rods and their corresponding equations. (Students can find the value of each colour by referring to the chart that shows the numeric values for the different colours of Cuisenaire rods.) For example, the Cuisenaire rods illustrated on the previous page and their corresponding equations might be recorded in the following way:
  - black = red + yellow ➛ 7 = 2 + 5
  - black = purple + light green ➛ 7 = 4 + 3
  - black = red + red + light green ➛ 7 = 2 + 2 + 3

Provide students with an opportunity to explore different combinations of Cuisenaire rods that have the same length and to record corresponding equations.

WORKING ON IT

Provide each pair of students with a spinner (made of EE2.BLM1: Combo Quest Spinner, a paper clip, and a pencil). Explain the activity:

- Students spin the spinner twice, select the Cuisenaire rods indicated by the spinner, and place the two rods end to end.
- Students find different combinations of Cuisenaire rods that have the same length as the rods indicated by the spinner. For example, if the spinner indicates the black rod and the red rod, students might find, among others, the following combinations:



- On a sheet of paper, students record the different combinations of Cuisenaire rods and their corresponding equations. For example, the Cuisenaire rods illustrated above and their corresponding equations might be recorded in the following way:
  - black + red = yellow + purple ➛ 7 + 2 = 5 + 4
  - black + red = light green + dark green ➛ 7 + 2 = 3 + 6
  - black + red = red + red + red + light green ➛ 7 + 2 = 2 + 2 + 2 + 3

As students find and record different combinations of rods and the corresponding equations, ask questions such as the following:

- "What is the value of the two rods indicated by the spinner?"
- "How can you find different combinations of rods that have the same value?"
- "What does this equation mean?"
- "How do you know that this part of the equation is equal to this other part?"
- "How could you prove that this equation is true?"

Provide students with an opportunity to repeat the activity of finding different combinations for two rods indicated by the spinner.

REFLECTING AND CONNECTING

Gather the students to discuss the activity. Pose the following questions to promote discussion:

- "What is an equation?"
- "How did this activity help you find equations?"
- "What are some of the equations you discovered?"
- "How could you use Cuisenaire rods to prove that 5 + 4 = 2 + 6 + 1?"
- "How could you use other manipulatives to represent this equation?"
- "For which two rods were you able to find many equivalent combinations of rods? Why?"

Have students respond to the following question in their math journals: "What is an equation?"

ADAPTATIONS/EXTENSIONS

For students who experience difficulties, simplify the activity by having them find combinations of rods that equal another rod. For example, students might find combinations that equal the brown rod (e.g., white and black, yellow and light green) and then record the corresponding equations (e.g., 8 = 1 + 7, 8 = 5 + 3).

Some students may not be ready to represent equations symbolically (e.g., 2 + 2 = 3 + 1). Have these students describe equalities orally and/or in written form (e.g., "7 and 2 is the same as 6 and 3"). Demonstrate how to record these ideas in symbolic notation by following a structure (i.e., __ + __ = __ + __).

For students who require a greater challenge, have them find different combinations that are equal to three Cuisenaire rods (e.g., find combinations of rods that equal the light green rod, the purple rod, and the yellow rod).

MATH LANGUAGE

- equal
- equation
- combination

ASSESSMENT

Assess students' understanding of equality and equations by observing students during the learning activity and by examining their math journal entries.

- *Are students able to find different equivalent combinations of Cuisenaire rods?*
- *Are students able to represent equations, using symbolic notation?*
- How well do students explain the meaning of *equation*?
- How well do students explain the meaning of the equal sign?

HOME CONNECTION

Send home EE2.BLM2: Equations Game. In this Home Connection activity, parents and students play a game that reinforces concepts about equations.

LEARNING CONNECTION 1

True or False?

Materials

- containers of interlocking cubes (1 container per pair of students)

Provide each pair of students with a container of interlocking cubes. Record the following number sentences on the board, one at a time.

- $7 = 3 + 4$
- $5 + 2 = 7 + 4$
- $8 = 4 + 2$
- $3 + 7 = 10$
- $5 + 4 = 9 + 1$
- $4 + 6 = 7 + 3$

Encourage students to use interlocking cubes to represent both sides of the number sentence, in order to determine whether each number sentence is true or false. For example, students might determine that the first number sentence is true by comparing the length of a row of 7 interlocking cubes and the length of a row composed of 3 red cubes and 4 blue cubes.

LEARNING CONNECTION 2

Agree or Disagree?

Materials

- counters

Record the following open-number sentence on the board: $4 + 2 = \_\_ + 5$

Explain that Sam, a student at another school, thinks that the answer is 6, since $4 + 2 = 6$. Ask: "Do you agree or disagree with Sam?"

Allow students time to examine the number sentence and to decide whether they agree or disagree with Sam.

Have students explain their decision to classmates. Have students use counters to demonstrate their thinking.

Provide other examples of open-number sentences to which Sam has correct or incorrect answers, and ask students whether they agree or disagree with Sam.

LEARNING CONNECTION 3

How Many Ways to Get to 11?

Materials

- *12 Ways to Get to 11* by Eve Merriam (New York: Alladin Paperbacks, 1996)
- a variety of manipulatives (e.g., interlocking cubes, counters, colour tiles)
- paper
- Kid Pix (ministry-licensed software)

Read *12 ways To Get To 11* aloud, if the book is available. This book presents situations that involve number combinations that add up to 11 (e.g., at the circus, 6 peanut shells and 5 pieces of popcorn).

Read the story a second time. Pause after reading each page, and ask students to explain how they could record a number sentence for the situation presented on the page. Record the students' responses on chart paper or the board. Discuss how students can use different equations (e.g., $9 + 2 = 11$, $11 = 9 + 2$, $2 + 9 = 11$) to represent the situation.

Note: If the book is unavailable, discuss different ways to use different objects (e.g., 4 markers and 7 pencils) to create sets of 11.

Arrange students in groups of 3. Have each group pick a number between 10 and 18. Instruct the students in each group to work together to find and record as many different equations for their number as possible. Provide students with a variety of manipulatives to help them find equations. Encourage students to find equations that involve more than two addends (e.g., 16 = 10 + 3 + 3).

Have groups create picture books similar to *12 Ways to Get to 11*. In their books, students create pictures (e.g., using Kid Pix) that illustrate equations for their chosen number. Have students entitle their book "___ Ways to Get to ___".

Have groups present their storybooks to the class.

LEARNING CONNECTION 4

Creating Equations With Colour Tiles

Materials

- colour tiles
- $8\frac{1}{2}$ in. × 11 in. sheets of paper (3 sheets per pair of students)

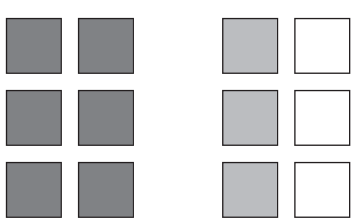
Place 7 red tiles on a sheet of paper, and ask students to count the tiles. Next, place 5 green tiles on another sheet of paper. Ask: "How many tiles do we need to add to the second paper so that the number of tiles is equal to the number of tiles on the first paper?" Ask students to explain how adding 2 tiles creates a quantity that is equal to 7. Add 2 yellow tiles to the second paper.

Relate the activity to an equation. On the board, record "7 = 5 + ___". Explain each part of the equation: "7" represents the number of tiles on one paper, "=" represents the idea that both sets of tiles need to be equal, "5" represents the original number of tiles on the second paper, and "+ ___" represents the number of tiles that need to be added to create equal sets.

Ask: "What number do we need to record in the blank? How do you know? How can you prove that both sides of the equation are equal?"

Repeat the activity, using other sets of tiles. Relate the equal sets to equations.

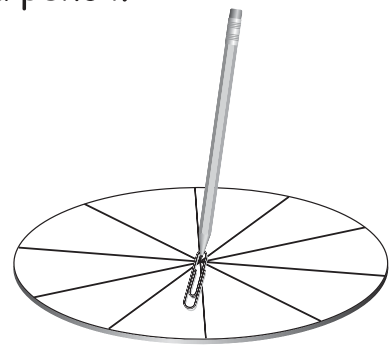
Provide each pair of students with colour tiles and three sheets of paper. Instruct students to use the tiles and two sheets of paper to find equal sets, and then to draw pictures of the tiles on a third sheet of paper. Ask students to record the corresponding equation for each picture.

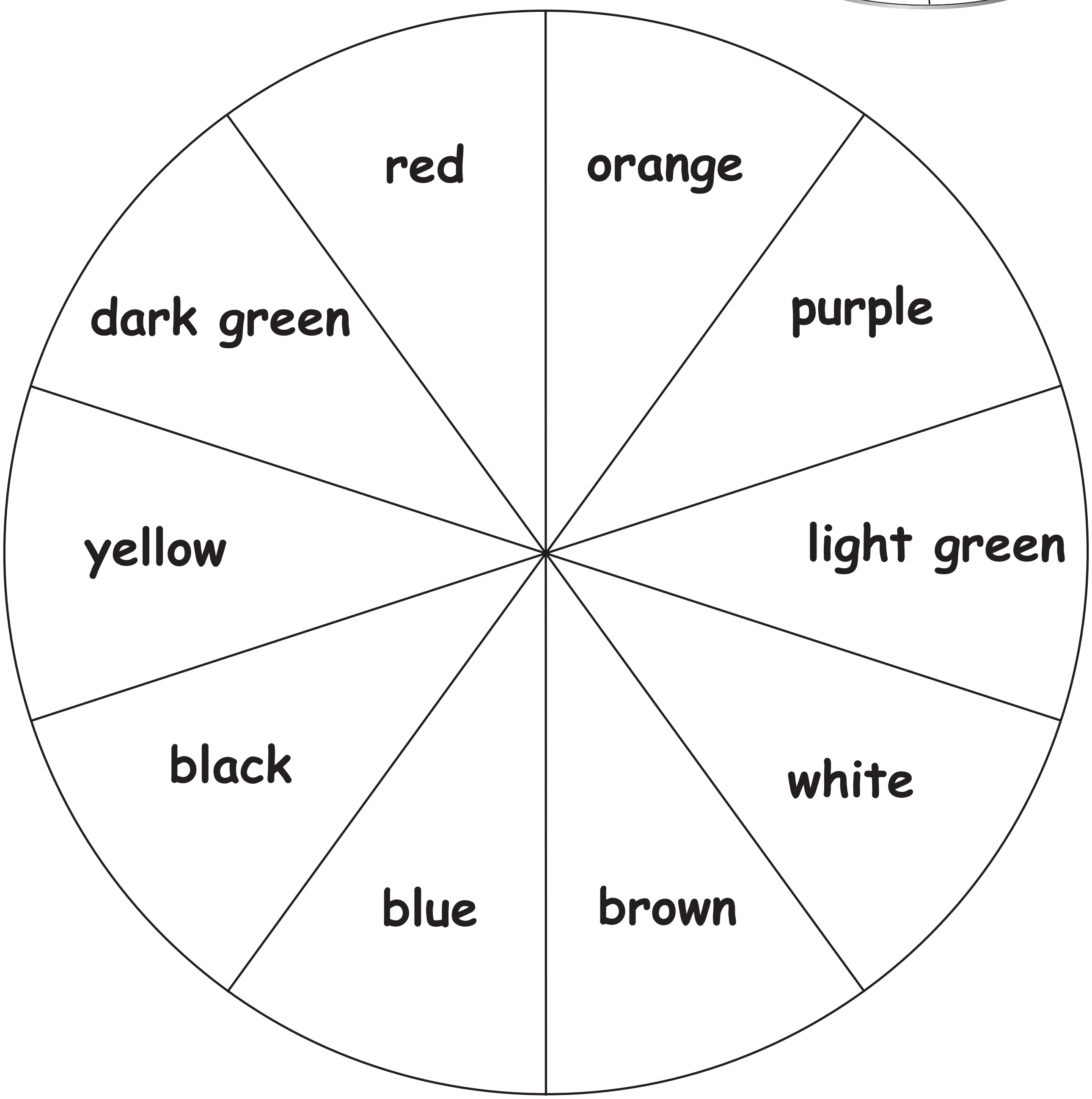


6 = 3 + 3

Combo Quest Spinner

Make a spinner, using this page, a paper clip, and a pencil.





Equations Game

Dear Parent/Guardian:

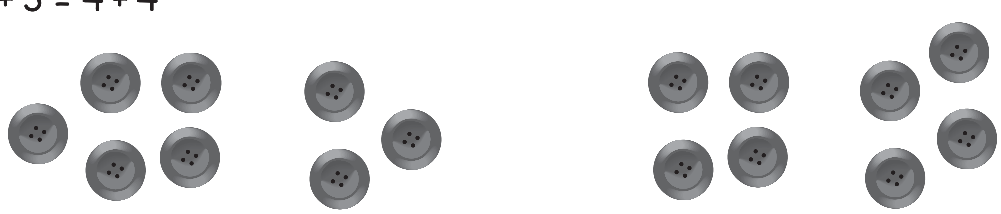
Our class has been learning about equations. For example, we investigated equations such as $3 + 4 = 2 + 5$, and we used concrete materials to show that $3 + 4$ is equal to $2 + 5$.

Here is a game that will reinforce ideas about equations with your child. For this game, you will need two number cubes (dice), paper, a pencil, and a collection of approximately 25 small objects (e.g., buttons, macaroni, beans).

How to Play the Game

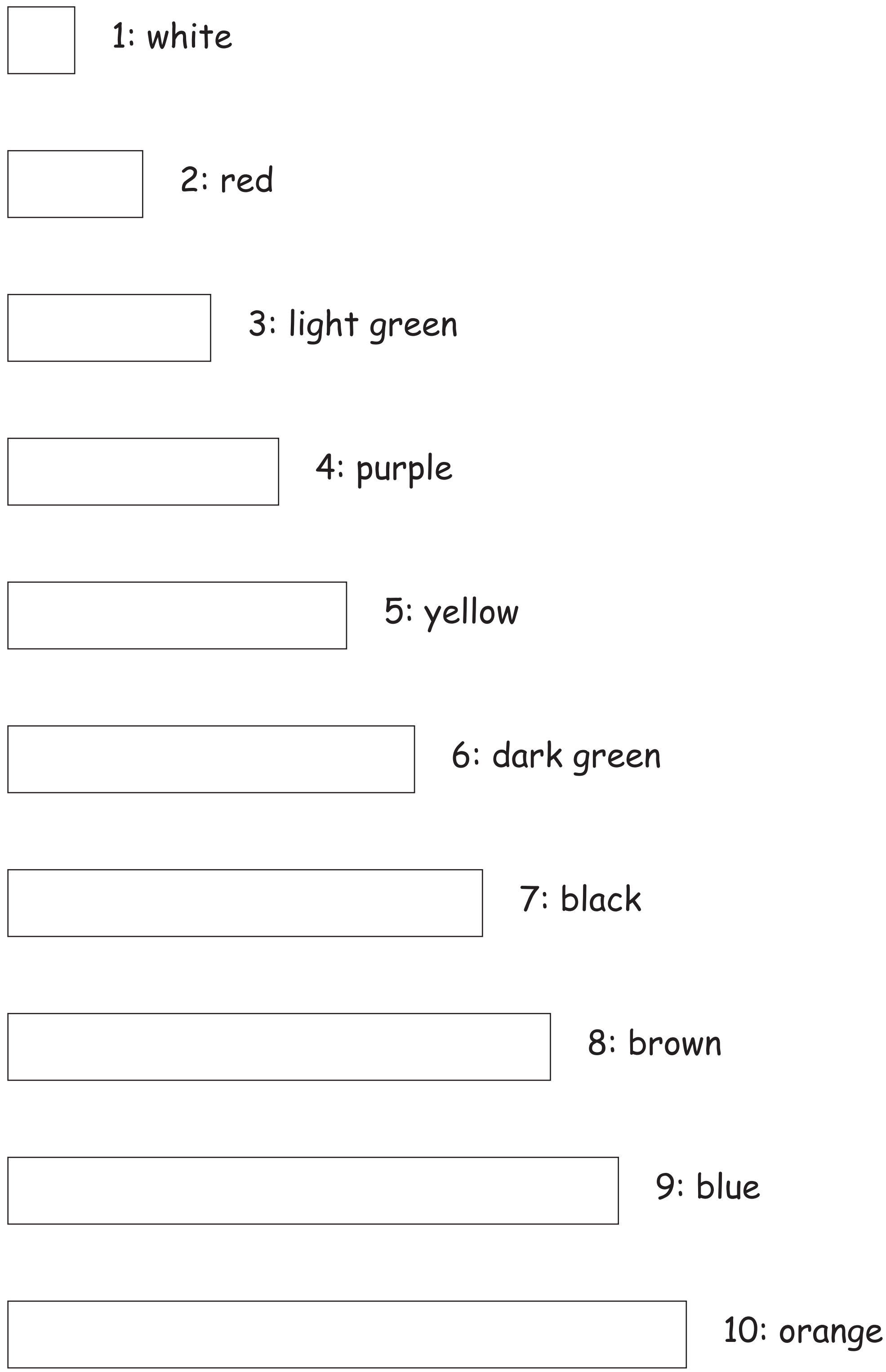
- Both players take a turn rolling two regular dice. Using the numbers shown on them, each player records on his or her paper an addition statement (without the answer). For example, if a player rolls 5 and 3, he or she records "$5 + 3 =$".
- After both players have recorded an addition sentence, they take turns rolling the regular dice. After each roll, the player determines whether the sum of the numbers on the regular dice is equal to the addition statement on his or her page. For example, if a player has recorded "$5 + 3 =$" and then rolls 1 and 4, the sum of 1 and 4 is not equal to $5 + 3$. However, if the player rolls 4 and 4, the sum of 4 and 4 is equal to $5 + 3$. The player then completes the equation on his or her page: $5 + 3 = 4 + 4$. The player must then use small objects to prove that the equation is correct (e.g., a group of 5 buttons plus a group of 3 buttons is equal to a group of 4 buttons plus a group of 4 buttons).

$5 + 3 = 4 + 4$



- The first player to prove that he or she has a correct equation wins the game.

Cuisenaire Rod Relationships



D. Grade 3 Learning Activities

Appendix Contents

Patty's Pattern Machines

BIG IDEA **Patterns and Relationships**

CURRICULUM EXPECTATIONS

Students will:

- extend repeating, growing, and shrinking number patterns;
- represent simple geometric patterns using a number sequence, a number line, or a bar graph (e.g., the given growing pattern of toothpick squares can be represented numerically by the sequence 4, 7, 10, . . . , which represents the number of toothpicks used to make each figure);

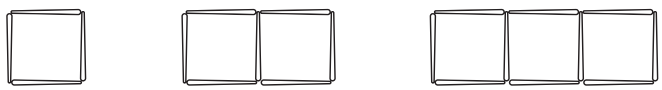


Figure 1 Figure 2 Figure 3

- demonstrate, through investigation, an understanding that a pattern results from repeating an action (e.g., clapping, taking a step forward every second), repeating an operation (e.g., addition, subtraction), using a transformation (e.g., slide, flip, turn), or making some other repeated change to an attribute (e.g., colour, orientation).

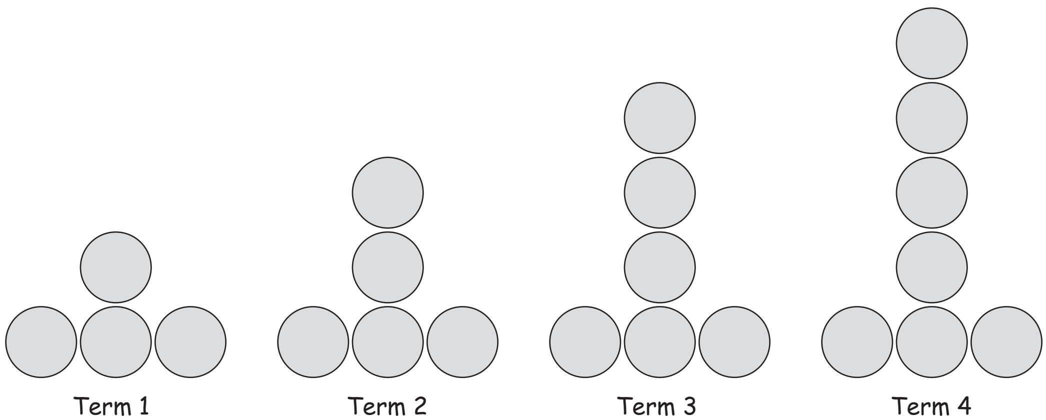
MATERIALS

- overhead projector
- blank overhead transparency
- overhead colour tiles
- overhead marker
- sheets of paper (1 per pair of students)
- pencils (1 per student)
- a large quantity of colour tiles
- half sheets of chart paper or large sheets of newsprint (1 per pair of students)
- markers (a few per pair of students)
- PR3.BLM1: Pattern Creator and Pattern Extender (1 per student)

ABOUT THE MATH

In the later primary grades, students investigate growing and shrinking patterns, using concrete materials. Their informal descriptions of how a pattern changes from term to term (e.g., "You add two more cubes each time" or "You take away a square each time") help them recognize how a growing or shrinking pattern can involve operations such as addition and subtraction. Seeing the relationships between a concrete pattern and the consistent, arithmetic change from term to term provides a basis for the development of algebraic thinking in later grades.

In Grade 3, students extend growing and shrinking patterns, using concrete materials, and represent numerically the increase or decrease of elements from term to term. For example, students might use counters to create the following growing pattern, and then represent the pattern by the number sequence 4, 5, 6, 7.



Using a number sequence to represent a growing or shrinking pattern helps students generalize the pattern (e.g., "Begin with 4 at term 1, and then add 1 at each new term"), and allows students to predict the number of elements at later terms (e.g., if the numbers representing the first four terms are 4, 5, 6, 7, then the number of elements at term 5 is 8, and the number of elements at term 6 is 9).

In Grade 3, students can also be introduced to tables as a way to organize numerical information related to growing and shrinking patterns.

Term Number	Number of Circles
1	4
2	5
3	6
4	7
5	?
6	?

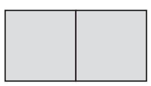
At this level, students may not recognize the functional relationship between the term number and the corresponding number of elements (e.g., the number of circles is always 3 more than the term number). However, students are able to recognize the recursive relationship (e.g., how the pattern grows or shrinks by 1 from term to term).

The following learning activity provides students with an opportunity to extend growing patterns, using concrete materials (i.e., colour tiles), and to represent the patterns numerically. The activity allows students to use a variety of strategies to determine the number of tiles in different terms of the pattern, and to find the total number of tiles in a given number of terms.

GETTING STARTED

Establish the context for the learning activity by discussing Patty and her pattern-making machines. Explain that at Patty's factory, machines produce patterns for other items that are decorated with patterns, such as wallpaper and fabrics.

Place a blank transparency on the overhead projector. Display two overhead colour tiles on the left side of the screen.

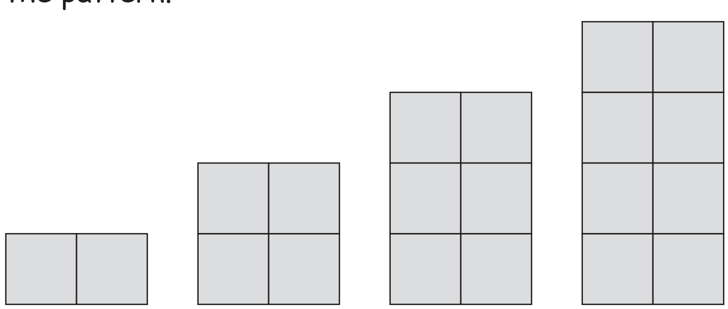


Explain that the two squares represent the first term in a pattern that is produced by one of Patty's machines. Below the tiles, record the following:

Term 1
2 squares

Continue to build and display the pattern (see the diagram below) by arranging overhead colour tiles on the overhead projector. Each time you add a new term to the pattern, ask: "What term in the pattern is this? How many squares are in this term?" Record the term number and number of squares below each term in the pattern.

Note: If the pattern is made of tiles of different colours, explain to the students that the machine chooses colours at random – the colour of the squares is not a feature of the pattern.



Term 1 2 squares | Term 2 4 squares | Term 3 6 squares | Term 4 8 squares

After displaying four terms of the pattern, ask the following questions:

- "Why is the pattern a growing pattern?"
- "How does the number of squares change from term to term?"
- "What number pattern do you see?"
- "How many squares in total are used to create the first four terms of the pattern? How did you find this total?"

Provide students with colour tiles, and instruct them to build the first four terms of the pattern according to the overhead display.

Next, have students predict the number of squares that will appear in the fifth and sixth terms. Ask them to explain their predictions. Direct students to construct the next two terms, and have them check their predictions.

Display the following table on chart paper or on the board.

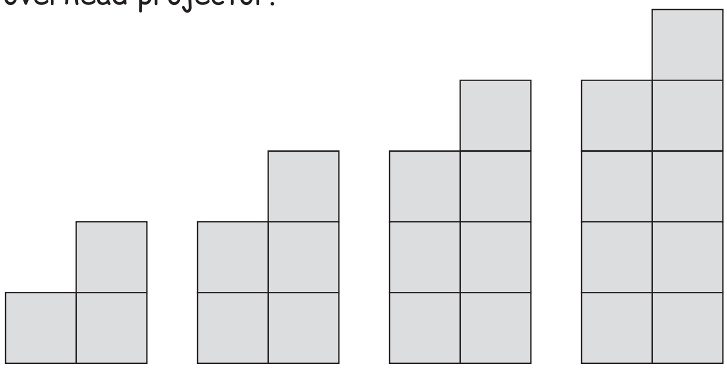
Term Number	Number of Squares
1	
2	
3	
4	
5	
6	

With input from the class, complete the second column of the table by recording the number of squares for each term. Have students describe patterns that they observe in the table.

Ask: "What is the pattern rule?" Have students summarize the rule, and record the rule below the table. For example, students might suggest the following pattern rule: "We added 2 more tiles each time we made a new term."

WORKING ON IT

Explain to students that another machine in Patty's factory produces a different pattern. Display the first four terms of the pattern by arranging overhead colour tiles on the overhead projector.



Explain that Patty wants the machine to create 10 terms, and that she needs to feed the required number of squares into the machine. Pose a problem: "How many squares will Patty need to feed into the machine so that it can create all 10 terms?"

Arrange students in pairs. Provide each pair with a sheet of paper and pencils, and make colour tiles available. Encourage students to discuss with their partner how they might solve the problem.

Observe the variety of strategies students use to solve the problem. Some students might use colour tiles to construct all 10 terms of the pattern, and then count the number of tiles. Others might use colour tiles to construct a few terms, discover the numeric pattern (i.e., 3, 5, 7, 9, . . .), and then find the total number of tiles by extending the number sequence. Observe, as well, how students record their work (e.g., drawing diagrams of the pattern terms; using tables to record numeric patterns). Ask students questions such as the following:

- "What geometric pattern do you see?"
- "What number pattern do you see?"
- "How does the number of squares change from term to term?"
- "What is the pattern rule?"
- "How can you find the total number of squares that Patty needs to feed into the machine?"

After pairs have solved the problem, provide them with a half sheet of chart paper or a large sheet of newsprint, and markers. Ask students to record how they solved the problem. Encourage them to record their solutions so that others will understand their thinking.

REFLECTING AND CONNECTING

Have pairs of students present their solutions to the class. Try to include a variety of strategies, so that students can observe different ways to solve the problem. Guide students' presentations by asking questions such as the following:

- "What strategy did you use to solve the problem?"
- "How did you use colour tiles? A diagram? A number sequence? A table?"
- "How were you able to extend a number sequence (complete a table) without having to use colour tiles to construct the entire pattern?"
- "How did you find the total number of tiles that Patty needs to feed into the machine?"

Post students' work following each presentation. After a variety of solutions have been presented, ask students to observe the different strategies and to compare the various methods. Ask:

- "Which strategies are similar? How are they alike?"
- "Which strategies involve using colour tiles? A diagram? A number sequence? A table?"
- "How could you use colour tiles (a diagram, a number sequence, a table) to help you find the total number of squares?"

Conclude the discussion by asking students to explain what they learned about different ways to solve problems involving growing patterns.

ADAPTATIONS/EXTENSIONS

For students who experience difficulty, simplify the problem by having them use colour tiles to continue the pattern, without requiring them to find the total number of tiles. The problem could also be simplified by providing a less complex pattern, or by having students find the total number of squares in the first 5 terms. Providing grid chart paper might help some students arrange the colour tiles.

Challenge students to create their own growing patterns for Patty's machine. Students can create patterns of varying complexity, and can determine the number of tiles needed for any number of terms.

MATH LANGUAGE

- term
- growing pattern
- geometric pattern
- number pattern
- number sequence
- table

ASSESSMENT

Observe students as they solve the problem, and later, when they present their solutions.

- How well do students identify and extend the growing pattern?
- How do students extend the pattern (e.g., using colour tiles, using a number sequence, using a table)?
- Do students need to use colour tiles to construct all 10 terms of the pattern, or are they able to generalize the pattern and use a number sequence to represent it?
- How well do students describe the geometric pattern? The numeric pattern?
- How well do students explain the pattern rule?
- Do students use an appropriate strategy to solve the problem (i.e., find the total number of tiles)?

HOME CONNECTION

Send home PR3.BLM1: Pattern Creator and Pattern Extender. In this Home Connection activity, students and their parents create and extend patterns.

LEARNING CONNECTION 1

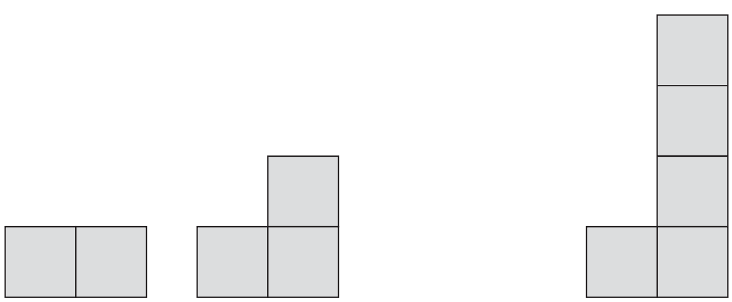
Find the Missing Term

Materials

– a variety of manipulatives (e.g., counters, pattern blocks, colour tiles, interlocking cubes)

Provide students with a variety of manipulatives (e.g., counters, pattern blocks, colour tiles, interlocking cubes). Instruct each student to use one of the manipulatives to create a growing or shrinking pattern. Ask the students to construct four terms of the pattern.

Next, ask the students to remove one of the terms and to leave a space in its place.



Arrange the students in pairs. Have the students examine their partner's pattern and reconstruct the missing term. Have the partners discuss whether the term was reconstructed correctly.

LEARNING CONNECTION 2

Draw the Missing Term

Materials

- PR3.BLM2: Draw the Missing Term (1 per student)
- a variety of manipulatives (e.g., counters, pattern blocks, colour tiles, interlocking cubes)

Provide each student with a copy of PR3.BLM2: Draw the Missing Term. Instruct students to examine each pattern and to draw the missing term. Encourage students to use manipulatives (e.g., counters, pattern blocks, colour tiles, interlocking cubes) to help them understand the pattern and find the missing term. (Note that the third pattern on the page is a shrinking pattern.)

After students have drawn the missing terms, provide them with an opportunity to describe the patterns to a partner.

LEARNING CONNECTION 3

Summer Chores

Materials

- PR3.BLM3: Summer Chores (1 per student)

Provide students with opportunities to identify and describe patterns on a calendar. For example, record regular class events, such as visits to the gym or library, on a calendar, and have students describe the resulting patterns. Students can also analyse and describe numeric patterns (e.g., numeric patterns in the rows and columns of a calendar).

Give each student a copy of PR3.BLM3: Summer Chores. Provide students with an opportunity to complete the patterns and to discuss them with a partner. Then engage the class in talking about the different patterns that students observe on the calendar.

LEARNING CONNECTION 4

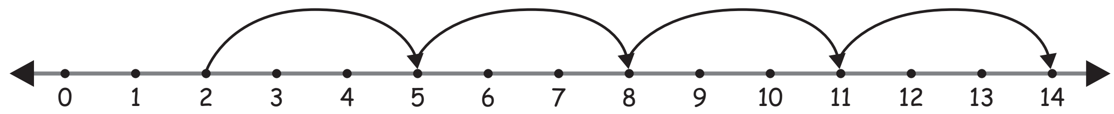
Creating Number Patterns

Materials

- sheets of paper (1 per student)
- pencils (1 per student)
- classroom number line

Provide each student with a sheet of paper and a pencil. Give instructions that allow students to create a number pattern (e.g., "Start at 2, and then keep adding 3 each time"). Have each student record the number pattern on his or her sheet of paper, and then discuss the results with the students.

Discuss how the number pattern can be represented on a number line.

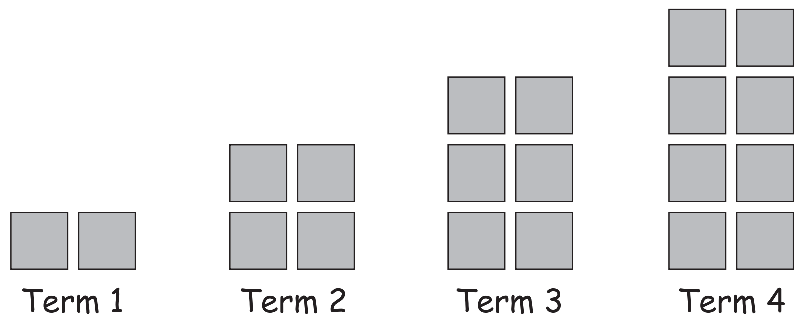


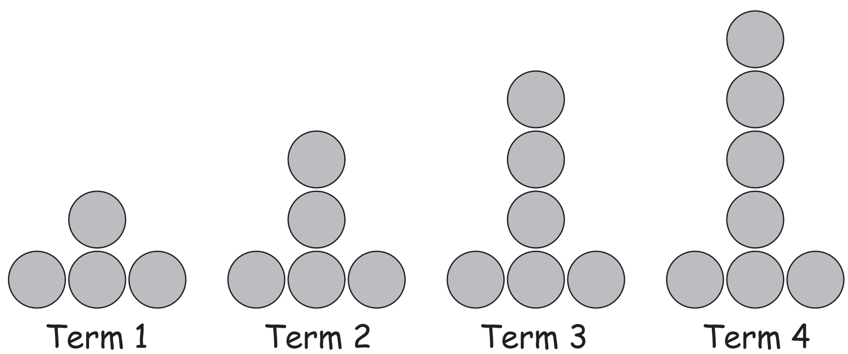
Provide students with opportunities to create number patterns involving addition or subtraction (e.g., “Start at 3, and then keep adding 5 each time” or “Start at 18, and then keep subtracting 3 each time”).

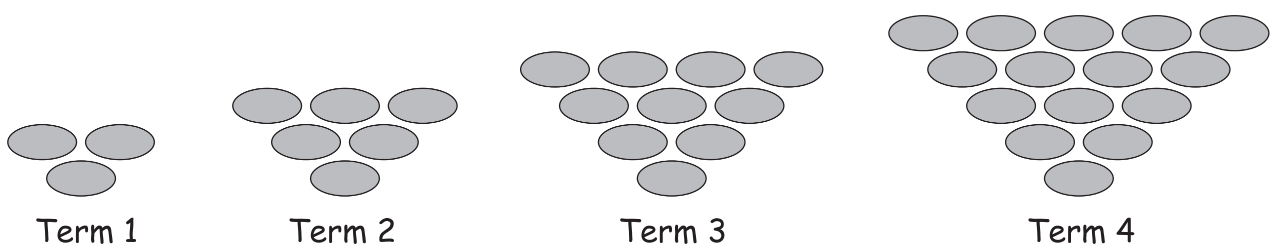
Pattern Creator and Pattern Extender

Dear Parent/Guardian:

We have been learning about growing patterns. Here are examples of growing patterns:





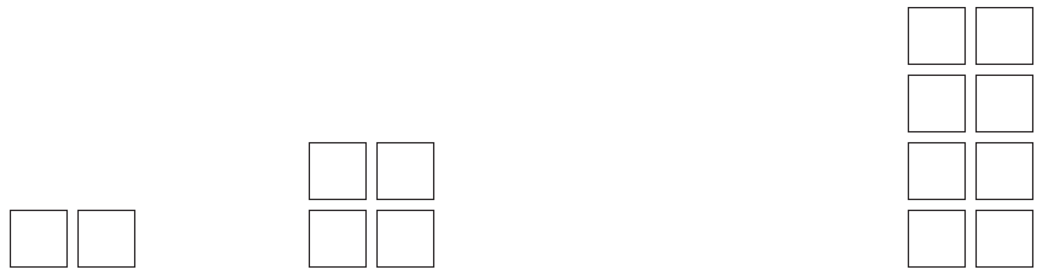


Create growing patterns with your child, using small items that you have at home (pennies, buttons, beans, macaroni). Take turns being the **pattern creator** and the **pattern extender**.

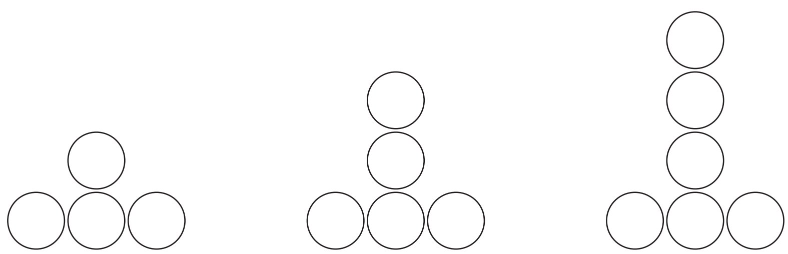
The **pattern creator** creates the first four terms of a growing pattern. The **pattern extender** examines the pattern and constructs term 5 and term 6 of the pattern. The pattern extender explains how the pattern grows from term to term.

Have fun creating and extending patterns!

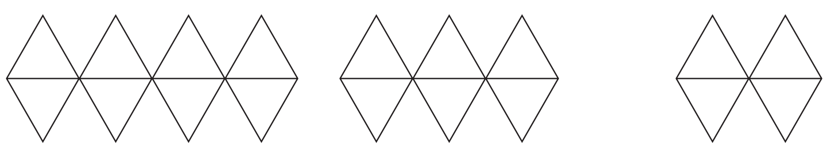
Draw the Missing Term



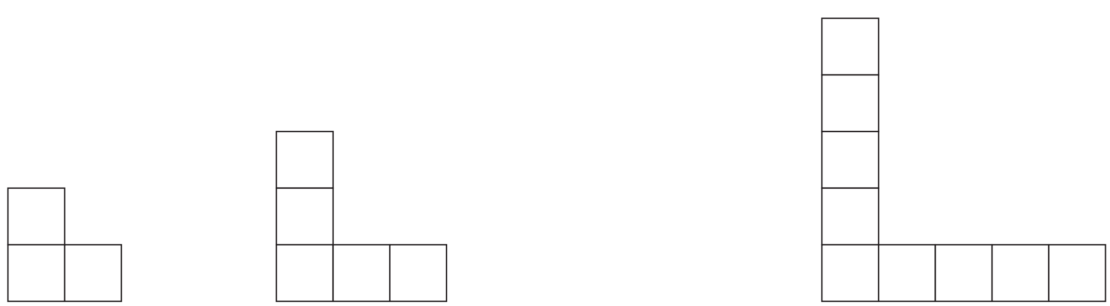
Term 1 Term 2 Term 3 Term 4



Term 1 Term 2 Term 3 Term 4



Term 1 Term 2 Term 3 Term 4



Term 1 Term 2 Term 3 Term 4

Describe each pattern to a partner.

Summer Chores

During the summer holidays, Jill enjoys helping with some chores at home. During the month of July, Jill sweeps the kitchen floor every 2nd day and washes the dishes every 3rd day.

Continue the pattern on the calendar to show when Jill sweeps the kitchen floor and washes the dishes.

July

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2 Sweep	3 Dishes	4 Sweep
5	6 Sweep Dishes	7	8 Sweep	9 Dishes	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Discuss with a partner: What patterns do you notice on the calendar?

Zeros and Ones Game

BIG IDEA Expressions and Equality

CURRICULUM EXPECTATIONS

Students will:

- identify, through investigation, the properties of zero and one in multiplication (i.e., any number multiplied by zero equals zero; any number multiplied by 1 equals the original number).

MATERIALS

- colour tiles (at least 25 per group of 3 students)
- number cubes (dice) (1 regular die per group of 3 students)
- EE3.BLM1: Zeros and Ones Score Card (1 per student)
- pencils
- math journals
- EE3.BLM2: Multiplying by 0 and 1 (1 per student)

ABOUT THE MATH

The development of multiplication concepts is a major focus of instruction in Grade 3. Students learn to represent multiplication, using a variety of concrete materials (e.g., creating equal groups of counters) and models (e.g., creating arrays). Through their exploration of the operation, students learn to recognize patterns in multiplication and begin to make generalizations about the operation. For example, students observe the effects of multiplication of a number by 0 and 1:

- Any number multiplied by 0 equals 0.
- Any number multiplied by 1 equals the original number.

It is important that students investigate the effects of 0 and 1 in multiplication, rather than rely on rules (e.g., "When you multiply a number by 1, it stays the same") that may have little meaning for students. In the following learning activity, students use colour tiles to explore the effects of multiplying numbers by 0 and 1. They then play a game that reinforces their understanding of the effects of 0 and 1 in multiplication.

GETTING STARTED

Provide each pair of students with a collection of colour tiles. On the board, record the following:

5 x 5
5 x 4
5 x 3
5 x 2
5 x 1
5 x 0

Challenge students to create arrays for the multiplication expressions, using colour tiles.

After students have created the arrays, ask the following questions:

- "What shapes do the arrays create?"
- "Why is the 5 x 5 array a square?"
- "How many tiles did you use in each arrangement?"
- "What do the numbers 5, 10, 15, 20, and 25 have in common?" (The numbers are multiples of 5.)
- "What does the 5 x 0 array look like? Why?"

Have students explain why it is not possible to create an array for 5 x 0. For example, students might explain that 5 rows (groups) of 0 is 0.

Repeat the activity, using other single-digit numbers. Emphasize the effects of the multiplication of numbers by 0 and 1. It is important that students use concrete materials to explore the effects of these operations before they play the Zeros and Ones game in the Working on It part of the learning activity.

WORKING ON IT

Explain the Zeros and Ones game:

- Students play the game in groups of 3. The players take turns.
- Each player rolls a regular die and records the number in the Number Rolled column on his or her personal copy of EE3.BLM1: Zeros and Ones Score Card.
- After the roll, the player chooses to multiply the number by 0 or 1.
- The player then records the result of the chosen operation in the Result column of his or her score card.
- After all players have completed 10 turns, they add the numbers in their Result columns to find their total scores. The player with the score closest to 15 wins the game. (The closest score may be over or under 15.)

Provide each group of students with a regular die, copies of EE3.BLM1: Zeros and Ones Score Card, and pencils. Encourage students to help one another perform the operations and record their results on the score cards.

Observe students as they play the game. Ask questions such as the following:

- "What is the result when you multiply a number by 0? By 1? Why?"
- "If you roll a 6 next time, which operation will you choose? Why?"
- "What is your strategy for getting close to 15?"
- "Which operation did you perform most often? Why?"

REFLECTING AND CONNECTING

After students have played the game, ask questions such as the following:

- "How did you try to get a score that was close to 15?"
- "What strategies did you use to play the game?"
- "Which operation did you choose often? Why?"
- "When did you choose to multiply a number by 0? By 1?"
- "What is the result of multiplying a number by 0? By 1? Why?"

As students discuss the different operations, invite them to use colour tiles to explain their thinking. It is important that students represent concepts about the operations concretely, rather than simply learn rules.

Record the following questions on the board:

- What is the result when you multiply a number by 0?
- What is the result when you multiply a number by 1?

Have students respond to the questions in their math journals. Encourage them to use diagrams and words to explain their thinking.

ADAPTATIONS/EXTENSIONS

Some students may be uncertain about the results of multiplying a number by 0 or 1. Have these students play the game with classmates who can use concrete materials (e.g., colour tiles) to help them understand the results of these operations.

Extend the game by providing other choices of operations (e.g., double the number rolled on the regular die).

MATH LANGUAGE

- multiply
- result
- operation

ASSESSMENT

Observe students as they play the game.

- How well do students understand the effects of multiplying a number by 0 and by 1?
- Do students use appropriate strategies for getting scores that are close to 15?
- How well do students calculate their scores?

Read students' math journal entries to assess how well they explain the effects of multiplying numbers by 0 and by 1.

HOME CONNECTION

Send home a copy of EE3.BLM2: Multiplying by 0 and 1. In this Home Connection game, players practise multiplying numbers by 0 and by 1.

LEARNING CONNECTION 1

Make It True!

Materials

- EE3.BLM3: Make It True! (1 per pair of student)
- number cubes (dice) (2 per pair of students)
- base ten blocks (a collection of tens rods and ones cubes per pair of students)
- calculators (1 per pair of students)

Provide each pair of students with a copy of EE3.BLM3: Make It True! and two regular dice. Explain the activity:

- Students roll the regular dice and create a two-digit number, using the numbers shown on them. For example, if the students roll a 3 and a 6, they may create either 36 or 63. They record the two-digit number in the box labelled "Roll 1".
- Next, students roll a single regular die and record the number shown in the box labelled "Roll 2".
- Students roll both regular dice to create another two-digit number, and record the number in the box labelled "Roll 3".
- Students work together to find the number for the right-most box that will make the statement true. They need to decide which operation to use (either addition or subtraction) and what number needs to go in the box in order to make the

equation true. Students may check their work, using base ten blocks, a calculator, and/or mental computation.

- Students create 5 different equations on the worksheet.

Observe students to assess how well they select an appropriate operation and find a correct number for each equation.

LEARNING CONNECTION 2

Making It Easier

Materials

– decks of playing cards with face cards removed (1 deck per pair of students)

– EE3.BLM4: Making it Easier (1 per pair of student)

– base ten blocks (a collection of tens rods and ones cubes per pair of students)

– calculators (1 per pair of students)

Arrange students in pairs. Explain the activity:

- Student A draws two cards and creates a two-digit number, using the digits shown on the cards. For example, if the student draws a 5 and a 6, he or she may create either 56 or 65. The student records the two-digit number in both squares of an equation on EE3.BLM4: Making it Easier.
- Student B draws two more cards, creates another two-digit number, and records the number in the circle of the equation.
- Students work together to decompose the two-digit number in the circle into parts. The following example shows how 34 might be decomposed into 30 and 4.

[56] + (34) = [56] + 30 + 4

- Students verify the equation, using base ten blocks, a calculator, and/or mental computation.

Discuss how decomposing an addend into parts helps in adding numbers mentally (e.g., to find 56 + 34, add 56 + 30 to get 86, and then add 86 + 4 to get 90).

LEARNING CONNECTION 3

Hidden Numbers

Materials

- EE3.BLM5: Hidden Numbers (1 per student)
- pencils (1 per student)

Provide each student with a copy of EE3.BLM5: Hidden Numbers and a pencil. Tell the students that some numbers are "hidden" under the squares and the circle on the page.

Discuss the first grid. Explain that the same number is under both squares and that a different number is under the circle. Describe how the grid works: the number under a square plus the number under the circle equals 6, the number under a square plus 3 equals 4, the number under the circle plus 3 equals 8, and so on.

Allow students time to find the hidden numbers in the first grid. Have students explain the hidden numbers and the strategies that they used to find them.

Have students find the hidden numbers in the other grids.

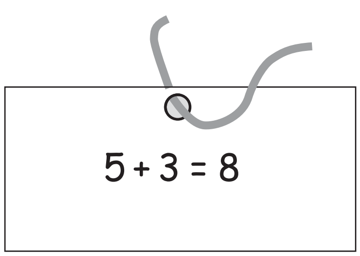
LEARNING CONNECTION 4

Find Your Partner

Materials

- EE3.BLM6a–d: Find Your Partner Equation Cards
- string
- paper punch
- interlocking cubes

Ahead of time, make necklaces, using string, a paper punch, and cards from EE3.BLM6a–d: Find Your Partner Equation Cards.



To begin the activity, model an addition sentence and its inverse subtraction sentence, using two colours of interlocking cubes (e.g., 5 red cubes + 3 blue cubes = 8 cubes, 8 cubes – 3 cubes = 5 cubes). On the board, record "5 + 3 = 8" and "8 – 3 = 5", and have students explain how the two sentences are related (e.g., 5 + 3 involves combining 5 and 3 to get 8, and 8 – 3 involves separating 3 from 8 to get 5). Repeat the process, using other combinations of interlocking cubes.

Play Find Your Partner. Place an addition or subtraction necklace around each student's neck, with the card hanging on the student's back so that he or she cannot see his or her own addition or subtraction sentence. (Place addition sentences around the necks of half the students, and the corresponding subtraction sentences around the necks of the other students.) Tell the students that they must try to find the person who has the corresponding addition or subtraction fact. For example, the student with 5 + 3 = 8 and the student with 8 – 3 = 5 need to find each other. Explain that, before finding the person with the corresponding fact, each student first needs to discover his or her own equation by asking classmates questions that they can answer with only yes or no. For example, a student might ask:

- "Do I have a subtraction number sentence?"
- "Is the difference in my number sentence 5?"
- "Is my number sentence 8 – 3 = 5?"

Have students walk around the classroom and ask questions of their classmates until each student is able to find his or her partner.

After the partners have been found, have students tell the number sentence that they think is on their card. Have the students reveal their cards to see if everyone found the right partner.

Zeros and Ones Score Card

	Number Rolled	Operation	Result
Example	5	x 1	5
Roll 1			
Roll 2			
Roll 3			
Roll 4			
Roll 5			
Roll 6			
Roll 7			
Roll 8			
Roll 9			
Roll 10			

Score __________

Multiplying by 0 and 1

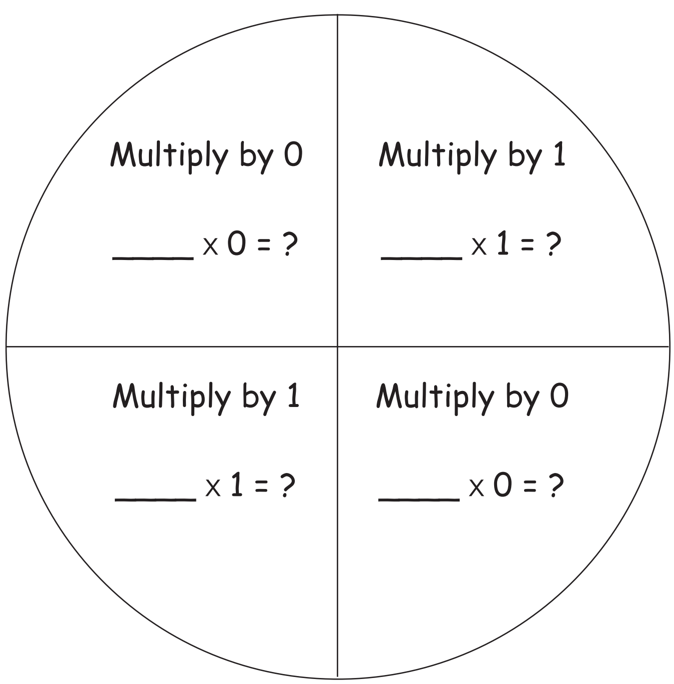
Dear Parent/Guardian:

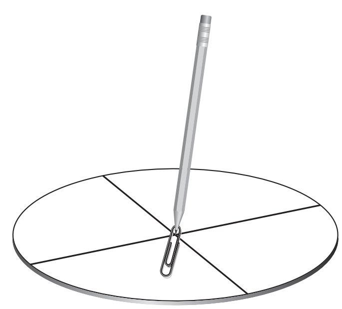
We have been learning about multiplying numbers by 0 and by 1. Here is a game for you to play with your child to reinforce these concepts. To play this game, you will need the following materials:

- a number cube (die)
- a spinner made of a paper clip, a pencil, and the spinner circle on this page
- paper and pencil for each player

How to Play the Game

- Player A rolls the regular die and spins the spinner.
- Player A multiplies the number on the regular die by either 0 or 1, according to the instruction shown on the spinner. The player records the answer on his or her paper.
- Player B takes a turn.
- Players continue to take turns until both players have had 10 turns.
- Players add the numbers on their paper. The player with the greater total wins the game.





Make It True!

Roll 1	+	Roll 2	=	Roll 3	+ or −	
Roll 1	+	Roll 2	=	Roll 3	+ or −	
Roll 1	+	Roll 2	=	Roll 3	+ or −	
Roll 1	+	Roll 2	=	Roll 3	+ or −	
Roll 1	+	Roll 2	=	Roll 3	+ or −	

Making It Easier

+ = + ____ + ____

+ = + ____ + ____

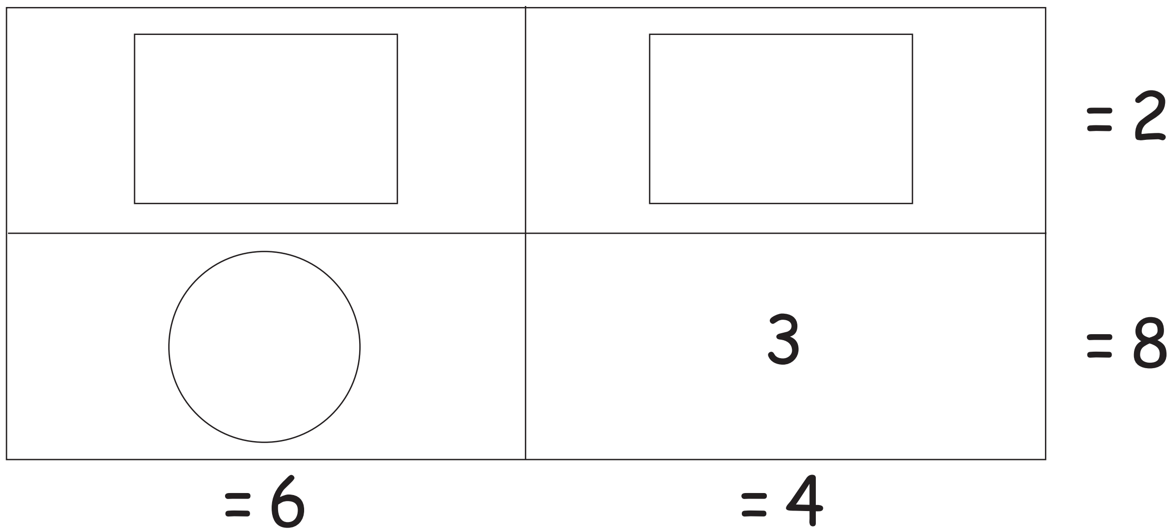
+ = + ____ + ____

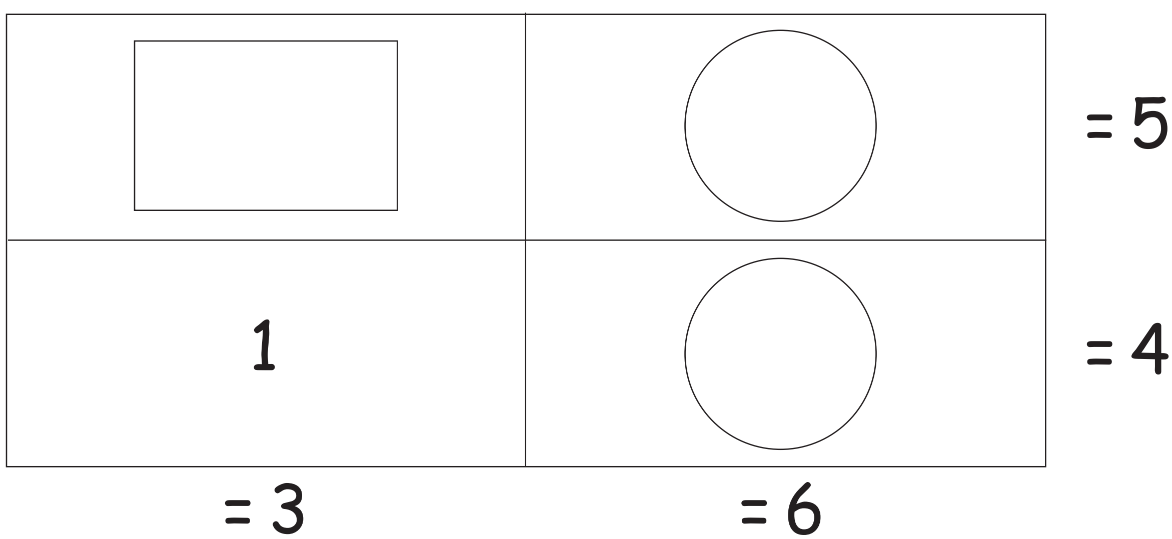
+ = + ____ + ____

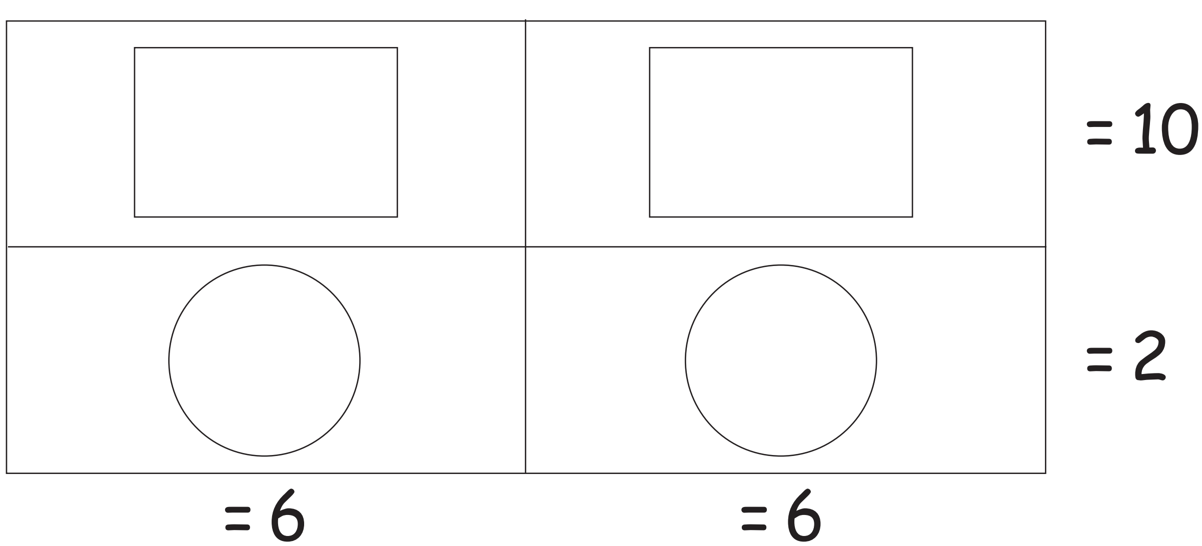
+ = + ____ + ____

What strategy did you use to make it easier to add the numbers?

Hidden Numbers







Find Your Partner Equation Cards

$5 + 3 = 8$	$8 - 3 = 5$
$4 + 6 = 10$	$10 - 6 = 4$
$9 = 5 + 4$	$9 - 4 = 5$
$6 + 2 = 8$	$8 - 2 = 6$
$10 = 5 + 5$	$10 - 5 = 5$

Find Your Partner Equation Cards

$5 = 2 + 3$	$5 - 2 = 3$
$4 + 2 = 6$	$6 - 2 = 4$
$10 = 3 + 7$	$10 - 3 = 7$
$7 + 1 = 8$	$8 - 1 = 7$
$2 + 2 = 4$	$4 - 2 = 2$

Find Your Partner Equation Cards

$9 = 7 + 2$	$9 - 7 = 2$
$3 + 4 = 7$	$7 - 4 = 3$
$1 + 2 = 3$	$3 - 2 = 1$
$4 = 3 + 1$	$4 - 3 = 1$
$4 + 4 = 8$	$8 - 4 = 4$

Find Your Partner Equation Cards

$1 + 5 = 6$	$6 - 5 = 1$
$8 + 2 = 10$	$10 - 2 = 8$
$5 = 1 + 4$	$5 - 1 = 4$
$1 + 1 = 2$	$2 - 1 = 1$
$5 + 2 = 7$	$7 - 2 = 5$

E. Correspondence of the Big Ideas and the Curriculum Expectations in Patterning and Algebra

Appendix Contents

Overall Expectations

KINDERGARTEN	GRADE ONE	GRADE TWO	GRADE THREE
Students will:			
• explore, recognize, describe, and create patterns, using a variety of materials in different contexts.	• identify, describe, extend, and create repeating patterns; • demonstrate an under-standing of the concept of equality, using concrete materials and addition and subtraction to 10.	• identify, describe, extend, and create repeating patterns, growing patterns, and shrinking patterns; • demonstrate an under-standing of the concept of equality between pairs of expressions, using concrete materials, symbols, and addition and subtraction to 18.	• describe, extend, and create a variety of numeric patterns and geometric patterns; • demonstrate an under-standing of equality between pairs of expressions, using addition and subtraction of one- and two-digit numbers.

Specific Expectations in Relation to the Big Ideas

KINDERGARTEN	GRADE ONE	GRADE TWO	GRADE THREE
Big Idea: Patterns and Relationships **Students will:**			
– identify, extend, reproduce, and create patterns through investigation, using a variety of materials *(e.g., attribute materials, pattern blocks, a hundreds chart, toys, bottle tops, buttons, toothpicks)* and actions *(e.g., physical actions such as clapping, jumping, tapping)*; – identify and describe informally the repeating nature of patterns in everyday contexts *(e.g., patterns in nature, clothing, floor tiles, literature, schedules)*, using oral expressions *(e.g., "goes before", "goes after", "morning, noon, and night", "the four seasons")* and gestures *(e.g., pointing, nodding)*.	– identify, describe, and extend, through investigation, geometric repeating patterns involving one attribute (e.g., colour, size, shape, thickness, orientation); – identify and extend, through investigation, numeric repeating patterns (e.g., 1, 2, 3, 1, 2, 3, 1, 2, 3, …); – describe numeric repeating patterns in a hundreds chart; – identify a rule for a repeating pattern (e.g., "We're lining up boy, girl, boy, girl, boy, girl."); – create a repeating pattern involving one attribute (e.g., colour, size, shape, sound); – represent a given repeating pattern in a variety of ways (e.g., pictures, actions, colours, sounds, numbers, letters).	– identify and describe, through investigation, growing patterns and shrinking patterns generated by the repeated addition or subtraction of 1's, 2's, 5's, 10's, and 25's on a number line and on a hundreds chart (e.g., the numbers 90, 80, 70, 60, 50, 40, 30, 20, 10 are in a straight line on a hundreds chart); – identify, describe, and create, through investigation, growing patterns and shrinking patterns involving addition and subtraction, with and without the use of calculators (e.g., $3+1=4$, $3+2=5$, $3+3=6$, …); – identify repeating, growing, and shrinking patterns found in real-life contexts (e.g., a geometric pattern on wallpaper, a rhythm pattern in music, a number pattern when counting dimes); – represent a given growing or shrinking pattern in a variety of ways (e.g., using pictures, actions, colours, sounds, numbers, letters, number lines, bar graphs);	– identify, extend, and create a repeating pattern involving two attributes (e.g., size, colour, orientation, number), using a variety of tools (e.g., pattern blocks, attribute blocks, drawings); – identify and describe, through investigation, number patterns involving addition, subtraction, and multiplication, represented on a number line, on a calendar, and on a hundreds chart (e.g., the multiples of 9 appear diagonally in a hundreds chart); – extend repeating, growing, and shrinking number patterns; – create a number pattern involving addition or subtraction, given a pattern represented on a number line or a pattern rule expressed in words; – represent simple geometric patterns using a number sequence, a number line, or a bar graph (e.g., the given growing pattern of toothpick squares can be represented

KINDERGARTEN	GRADE ONE	GRADE TWO	GRADE THREE
Big Idea: Patterns and Relationships (*cont.*) **Students will:**			
		– create growing or shrinking patterns; – create a repeating pattern by combining two attributes (e.g., colour and shape; colour and size); – demonstrate, through investigation, an understanding that a pattern results from repeating an operation (e.g., addition, subtraction) or making a repeated change to an attribute (e.g., colour, orientation).	numerically by the sequence 4, 7, 10, …, which represents the number of toothpicks used to make each figure);  Figure 1 Figure 2 Figure 3 – demonstrate, through investigation, an understanding that a pattern results from repeating an action (e.g., clapping, taking a step forward every second), repeating an operation (e.g., addition, subtraction), using a transformation (e.g., slide, flip, turn), or making some other repeated change to an attribute (e.g., colour, orientation).
Big Idea: Expressions and Equality **Students will:**			
	– create a set in which the number of objects is greater than, less than, or equal to the number of objects in a given set; – demonstrate examples of equality, through investigation, using a “balance” model; – determine, through investigation using a “balance” model and whole numbers to 10, the number of identical objects that must be added or subtracted to establish equality.	– demonstrate an understanding of the concept of equality by partitioning whole numbers to 18 in a variety of ways, using concrete materials (e.g., starting with 9 tiles and adding 6 more tiles gives the same result as starting with 10 tiles and adding 5 more tiles); – represent, through investigation with concrete materials and pictures, two number expressions that are equal, using the equal sign (e.g., “I can break a train of 10 cubes into 4 cubes and 6 cubes. I can also break 10 cubes into 7 cubes and 3 cubes. This means $4 + 6 = 7 + 3$.”); – determine the missing number in equations involving addition and subtraction to 18, using a variety of tools and strategies (e.g., modelling with concrete materials, using	– determine, through investigation, the inverse relationship between addition and subtraction (e.g., since $4 + 5 = 9$, then $9 - 5 = 4$; since $16 - 9 = 7$, then $7 + 9 = 16$); – determine the missing number in equations involving addition and subtraction of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator); – identify, through investigation, the properties of zero and one in multiplication (i.e., any number multiplied by zero equals zero; any number multiplied by 1 equals the original number); – identify, through investigation, and use the associative property of addition to

KINDERGARTEN	GRADE ONE	GRADE TWO	GRADE THREE
Big Idea: Expressions and Equality (*cont.*) Students will:			
		guess and check with and without the aid of a calculator); – identify, through investigation, and use the commutative property of addition (e.g., create a train of 10 cubes by joining 4 red cubes to 6 blue cubes, or by joining 6 blue cubes to 4 red cubes) to facilitate computation with whole numbers (e.g., "I know that 9 + 8 + 1 = 9 + 1 + 8. Adding becomes easier because that gives 10 + 8 = 18."); – identify, through investigation, the properties of zero in addition and subtraction (i.e., when you add zero to a number, the number does not change; when you subtract zero from a number, the number does not change).	facilitate computation with whole numbers (e.g., "I know that 17 + 16 equals 17 + 3 + 13. This is easier to add in my head because I get 20 + 13 = 33.").

Glossary

Note: Words and phrases in boldface italics in the following definitions are also defined in the glossary.

algebra. A branch of mathematics that uses letters, symbols, and/or characters to represent numbers and to express mathematical ***relationships***.

algebraic reasoning. Thinking that involves generalizing and modelling ***patterns*** symbolically.

assessment. The ongoing, systematic gathering, recording, and analysis of information about a student's achievement, using a variety of strategies and tools. Its intent is to provide the teacher (and the student, where appropriate) with feedback that can be used to improve programming.

associative property. A property of addition and multiplication that allows the numbers being added or multiplied to be grouped differently without changing the result. For example, (7 + 9) + 1 = 7 + (9 + 1), and (7 x 4) x 5 = 7 x (4 x 5). Using the associative property can simplify computation. The property does not generally hold for subtraction or division.

attribute. A qualitative or quantitative characteristic of a shape, an object, or an occurrence; for example, colour, size, thickness, or number of sides.

attribute blocks. Learning tools that help students learn about shapes, sorting, patterning, geometric properties, and so on. The standard set of attribute blocks (60 blocks) includes five shapes (rectangle, square, circle, triangle, hexagon); each shape comes in three colours (red, yellow, blue), two sizes (large, small), and two thicknesses (thick, thin).

balance. A device consisting of two pans, buckets, or platforms supported at opposite ends of a balance beam. A balance is used to compare and measure ***masses*** of objects.

big ideas. In mathematics, the important concepts or major underlying principles. For example, the big ideas for Kindergarten to Grade 3 in the Patterning and Algebra strand of the Ontario curriculum are ***patterns*** and ***relationships***, and ***expressions*** and ***equality***.

commutative property. A property of addition and multiplication that allows the numbers to be added or multiplied in any order without affecting the sum or product. For example, 2 + 3 = 3 + 2, and 3 x 2 = 2 x 3. Using the commutative property can simplify computation. The property does not generally hold for subtraction or division.

concept. *See* ***mathematical concept***.

concrete material. *See* ***manipulative***.

concrete pattern. ***Patterns*** that are represented by ***concrete materials*** (e.g., pattern blocks, interlocking cubes, colour tiles).

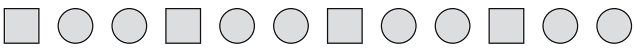
conjecture. A guess or prediction, based on observed ***patterns***, that has not been proven to be either true or false.

context. The environment, situation, or setting in which an event or activity takes place. Real-life settings often help students make sense of mathematics.

core. (Also called "stem".) The basic string of ***elements*** that repeats in a ***pattern***. In an ABB-ABB-ABB pattern, the core is ABB.

Cuisenaire rods. Commercially produced manipulatives consisting of a set of rectangular rods of different lengths, in which each length is a different colour.

element. A specific item (e.g., object, shape, number) within a ***pattern***. The elements in the following pattern are a circle and a square.



equality. The state of being equal. An equality can be represented by an ***equation*** (e.g., 4 + 5 = 6 + 3). *See also* ***inequality***.

equal sign. A mathematical symbol (i.e., =) used to indicate ***equality***. The ***expression*** on the left of the equal sign is equivalent to the expression on its right (e.g., 3 + 1 = 2 + 2).

equation. A mathematical statement in which equivalent ***expressions*** are on either side of an ***equal sign***.

expectations. The knowledge and skills that students are expected to learn and to demonstrate by the end of every grade or course, as outlined in the Ontario curriculum documents for the various subject areas.

expression. A numeric or algebraic representation of a ***quantity***. An expression may include numbers, ***variables***, and operations; for example, 3 + 7, $2x - 1$.

extension. A learning activity that is related to a previous one. An extension can involve a task that reinforces, builds on, or requires application of newly learned material.

functional relationship. A mathematical ***relationship*** between two sets of numbers, where every number value in one set corresponds to exactly one number value in the other set. In the following example, the functional relationship between the number of bicycles and the number of wheels can be expressed as "x 2".

Number of Bicycles	Number of Wheels
1	2
2	4
3	6
4	8

x 2

generalization. The process of determining a general rule or making a conclusion based on the observation of given examples.

geometric pattern. An arrangement of geometric shapes or figures that repeats.

growing pattern. A pattern that involves a progression (e.g., growth of ***elements***) from term to term (e.g. A, AA, AAA, AAAA).

inequality. The state of not being equal. An inequality can be represented by an ***expression*** (e.g., 4 + 5 ≠ 6 + 2). *See also* ***equality***.

investigation. An instructional activity in which students pursue a problem or an exploration. Investigations help students develop problem-solving skills, learn new concepts, and apply and deepen their understanding of previously learned concepts and skills.

learning styles. Different ways of learning and processing information. For instance, visual learners need to see visual representations of concepts. Auditory learners learn best through verbal instructions and discussions, and by talking things through and listening to what others have to say. Tactile/kinaesthetic learners learn best through a hands-on approach, and by exploring the physical world around them.

manipulatives. (Also called "concrete materials".) Objects that students handle and use in constructing or demonstrating their understanding of ***mathematical concepts*** and skills. Some examples of manipulatives are counters, interlocking cubes, and colour tiles.

mass. The amount of matter in an object; usually measured in grams and kilograms. In the primary grades, students use ***balances*** and non-standard units, such as metal washers and marbles, to investigate mass.

mathematical concept. A fundamental understanding about mathematics that a student develops within problem-solving ***contexts***.

mode. The general manner in which a ***pattern*** is represented. Students can represent patterns in a variety of modes (e.g., using ***manipulatives***, using actions, using shapes, using letters.)

multiple. The product of a given whole number multiplied by any other whole number. For example, 4, 8, 12, . . . are multiples of 4.

numeric patterns. Patterns composed of numbers (e.g., 5, 10, 15, 20, . . .).

pattern. A repeated design or recurring sequence.

pattern blocks. Commercially produced learning tools consisting of green triangles, orange squares, tan rhombuses and larger blue rhombuses, red trapezoids, and yellow hexagons.

pattern rule. A description of how a ***pattern*** repeats, grows, or shrinks, based on a ***generalization*** about the ***pattern structure***. For example, the pattern rule for the ***growing pattern*** 3, 7, 11, 15, . . . might be expressed as "begin at 3, and repeatedly add 4". *See also* **repeating pattern**, **growing pattern**, and **shrinking pattern**.

pattern structure. The composition of a ***pattern***. A pattern structure can often be represented by a series of letters. For example, a pattern involving triangle – square – square – triangle – square – square – triangle – square – square can be represented as ABB, ABB, ABB.

problem solving. Engaging in a task for which the solution is not obvious or known in advance. To solve the problem, students must draw on their previous knowledge, try different strategies, make connections, and reach conclusions. Learning to solve problems by inquiry or ***investigation*** is very natural for young students.

quantity. The "howmuchness" of a number. An understanding of quantity helps students estimate and reason with numbers, and is an important prerequisite to understanding place value, the operations, and fractions.

recursive relationship. A description that tells how a ***growing pattern*** or ***shrinking pattern*** changes from one term to the next. In the following example, the recursive relationship can be expressed as " + 5".

Day	Money Earned ($)	
1	5	
		+ 5
2	10	
		+ 5
3	15	
		+ 5
4	20	

relationship. In mathematics, a connection between ***mathematical concepts***, or between a ***mathematical concept*** and an idea in another subject or in real life. As students connect ideas they already understand with new experiences and ideas, their understanding of mathematical relationships develops.

repeating pattern. A ***pattern*** in which a ***core*** repeats continuously (e.g., AAB, AAB, AAB, . . .).

representation. The use of ***manipulatives***, diagrams, pictures, or symbols to model a ***mathematical concept*** or a real-world ***context*** or situation.

shrinking pattern. A ***pattern*** that involves a regression (e.g., a decrease in the number of elements) from term to term (e.g. AAAA, AAA, AA, A).

stem. *See* **core**.

strand. A major area of knowledge and skills. In the Ontario mathematics curriculum for Grades 1–8, there are five strands: Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning and Algebra, and Data Management and Probability.

variable. A shape, letter, or symbol used to represent an unknown ***quantity***, a changing value, or an unspecified number (e.g., $a \times b = b \times a$).