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Patterning and Algebra Learning Activities – Grade 6

You’re a Winner!

Strand: Patterning and Algebra, Grade 6

Big Ideas: Patterns and relationships

Overview

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In this learning activity, students investigate growing patterns that result in triangular numbers (numbers such as 1, 3, 6, and 10, which can be expressed as triangular patterns of dots). They also investigate the patterns in finding the sums of triangular numbers. The “You’re a Winner!” activity involves the analysis of a highly unusual method of delivering a collection of miniature toys to the first-prize winner at a fun fair:

Day 1 – 1 chipmunk

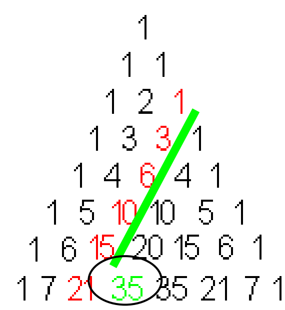
Day 2 – 1 chipmunk and 2 blue jays

Day 3 – 1 chipmunk, 2 blue jays, 3 puppies

The delivery would continue each day in the same manner until the 10th day.

Day 10 – 1 chipmunk, 2 blue jays, 3 puppies, 4 kittens, 5 butterflies, 6

The mathematical context is Pascal’s Triangle (shown on the right).



Prior to this learning activity, students should have had some experience with representing patterns and using tables, diagrams, and manipulatives.

The students will be challenged to choose to receive 25 toys immediately or accept the unusual deliver method.

Curriculum Expectations

Overall Expectation:

- describe and represent relationships in growing and shrinking patterns (where the terms are whole numbers), and investigate repeating patterns involving rotations.

Specific Expectations

- make tables of values, for growing patterns given pattern rules, in words (e.g., start with 3, then double each term and add 1 to get the next term), then list the ordered pairs (with the first coordinate representing the term number and the second coordinate representing the term) and plot the points in the first quadrant, using a variety of tools (e.g., graph paper, calculators, dynamic statistical software);
- determine the term number of a given term in a growing pattern that is represented by a pattern rule in words, a table of values, or a graph;
- describe pattern rules (in words) that generate patterns by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term (e.g., for 1, 3, 5, 7, 9,…, the pattern rule is "start with 1 and add 2 to each term to get the next term"), then distinguish such pattern rules from pattern rules, given in words, that describe the general term by referring to the term number (e.g., for 2, 4, 6, 8,…, the pattern rule for the general term is "double the term number");
- determine a term, given its term number, by extending growing and shrinking patterns that are generated by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term.

About the Learning Activity

Time: 120 minutes

Materials

- **PA.BLM6a.1: Anticipation Guide**
- **PA.BLM6a.2: Pascal's Triangle**
- **PA.BLM6a.3: Problems Carousel**
- **PA.BLM6a.4: Home Connection**
- chart paper
- markers
- coloured tiles, interlocking cubes, relational rods, pattern blocks, and/or beads/buttons
- calculators

Math Language

- triangular numbers, Pascal's Triangle, sequence, consecutive numbers, sequential, spatial, temporal, linguistic

Instructional Grouping: "table groups" (approximately 4 students)

About the Math

How to generate triangular numbers

- Start with 1, add 2 to get the second number, add 3 to get the third, add 4 to get the fourth, and so on.
- Start with 1 dot, add 2 dots below it to form a triangle of 3 dots, then add 3 dots below that to form a triangle of 6 dots, then add 4 dots below that to form a triangle of 10 dots, and so on.

$$1$$
$$1+2 = 3$$
$$1+2+3 = 6$$
$$1+2+3+4 = 10$$

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      * *    * *        * *
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How to find the value of a triangular number

If we are asked, for example, “What is the 20th triangular number?”, we can find its value without first determining the triangular numbers that precede it. From the patterns above we can see that the 20th triangular number is the sum of 1+2+3+4+ …+18+19+20. How can we find this sum without adding all these numbers together? One way is to notice that 1+20 = 21, 2+19 = 21, 3+18 = 21, and so on. There are 10 such pairs, so the 20th triangular number is 10 x 21 = 210. Another way is to notice that a triangular number and its copy can fit together to form a rectangle.

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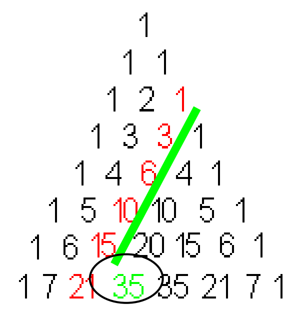
The 4th triangular number is 4 x 5 ÷ 2 = 10.

The 20th triangular number is 20 x 21 ÷ 2 = 210.

How to find the sum of the first 20 triangular numbers

The first 10 triangular numbers have 10 ones, 9 twos, 8 threes, and so on. So the 10th triangular number is equal to 10×1 + 9×2 + 8×3 + 7×4 + … + 3×8 + 2×9 + 1×10. The sum is also found in Pascal’s Triangle (see description below). The diagram shows that the sum of the first five triangular numbers (1+3+6+10+15) is 35 (see http://www.mathematische-basteleien.de/triangularnumber.htm).



Pascal’s Triangle

Pascal’s Triangle is named after Blaise Pascal, the French mathematician who studied it. Interestingly, this pattern was known in China long before Pascal’s time. Notice how Pascal’s Triangle grows:

- the numbers on the outer diagonals have a value of 1;
- each of the other numbers is the sum of the two numbers directly above it (for example, the 6 in the 5th row is the sum of the two 3s above it).

Pascal’s Triangle has many applications in mathematics and contains within itself many interesting patterns, such as triangular numbers.

Getting Started – Warm Up (Part 1 of a 3 part lesson)

Activating prior knowledge

Divide the class into groups of up to four students each. Tell students that they are going to be exploring and generating patterns. Have them complete the "Before" side of the Anticipation Guide (**PA.BLM6a.1**) as a way of activating prior knowledge and identifying common misconceptions. Then have them brainstorm applications of patterns in the real world. Patterns may be categorized as sequential (playing cards), spatial (buildings), temporal (calendars), and linguistic (spelling rules).

Working on It – (Part 2 of the 3 part lesson)

Introduce this scenario to the whole class:

"At a community fair, a student wins first prize – a collection of miniature toys. He has a choice to have 25 toys immediately or have toys delivered everyday to his home for the next ten days. For the next ten days the toys would be delivered to the student's home in a most unusual way. On the first day the winner's package contains 1 chipmunk; on the second day the package contains 2 blue jays and 1 chipmunk; on the third day the package contains 3 puppies, 2 blue jays, and 1 chipmunk; on the fourth day the package contains 4 kittens, 3 puppies, 2 blue jays, and 1 chipmunk. The same pattern continues until, on the tenth day, the package contains 10 caterpillars, 9 ladybugs, 8 goldfish, 7 rabbits, 6 ducklings, 5 butterflies, 4 kittens, 3 puppies, 2 blue jays, and 1 chipmunk. And the question is: "Which way of receiving the prizes is the best choice for the winner?

Understanding the problem

Have students think about what the solution(s) might look like and have them rephrase the question. Ensure that manipulatives are available and invite students to use them to model the problem. Tell students that their groups will be required to display their solutions on chart paper and explain their processes to other groups and eventually to the class.

Make a Plan

Circulate and ensure that each group understands the task and has a plan of action. If groups are having trouble developing strategies, allow one member to "phone a friend" (i.e., visit another group to observe other strategies). Understanding the question and developing an appropriate strategy will be challenging for some students. You might prompt students' thinking by asking:

- What could you use in the classroom to model the problem?
- What strategies have you considered?
- How might you organize the information?

Looking Back

After students have solved the problem and decided what choice the winner should make, gather the students together for a whole class discussion. Each group of students should share their findings with one other group before presenting to the whole class.

Select three or four students who have different representations or strategies to explain their chart and their thinking. Some anticipated student responses might include:

Create a table of values. The problem can be clarified for some students if they create a table of values such as the one below.

Day	Toys
1	chipmunk
2	chipmunk, blue jay, blue jay
3	chipmunk, blue jay, blue jay, puppy, puppy, puppy, puppy

Model using concrete materials. Students may choose to use beads, tiles, cubes, and so on, to build a day-by-day model on their desks. They might also use the letters A through J to represent the 10 types of toys. Thus, the toys received on the fifth day would be ABBCCCDDDDEEEEE.

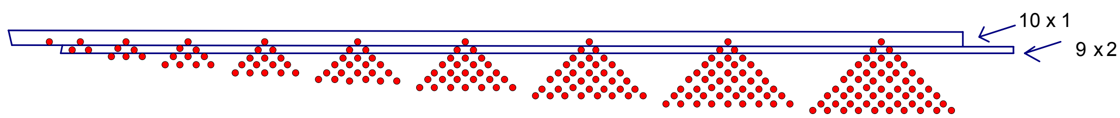
Use numerical representation. Some students might develop an algorithm, such as:

> **Note:** Students are unlikely to use brackets in the first algorithm.

day1 +day1+day2+day1+day2+day3 = 1+(1+2)+(1+2+3) **or**

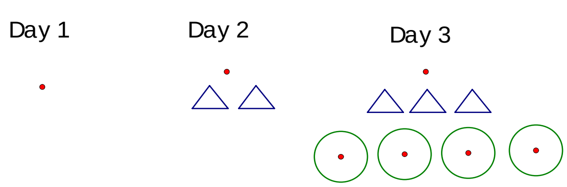
total day1+total day2+total day3 = 1+3+6+10… **or**

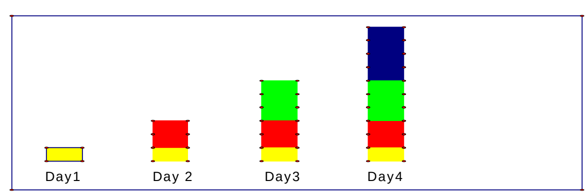
The first 10 triangular numbers have 10 ones, 9 twos, 8 threes, and so on. Therefore, the 10th triangular number is equal to $10\times1 + 9\times2 + 8\times3 + 7\times4 + \ldots + 3\times8 + 2\times9 + 1\times10$, as shown below.



> **Teacher note:** Recognizing that 10 x 1 + 9 x 2 + 8 x 3+…+2 x 9 + 1 x 10 is an equivalent representation of adding together the total number of toys each day to get the sum of the first 10 days indicates a developmental leap in a student's understanding of patterning concepts.

Draw a diagram. Some students may draw symbols to represent their solution. Other students may use graph paper to draw their representation. Encourage groups to clearly explain their thought processes on chart paper. They should use numbers, pictures, and words.





Reflecting and Connecting – (Part 3 of a 3 part lesson)

During students' presentations, avoid implying that some strategies are better than others. Encourage students to consider the range of strategies and to try and make sense of each representation. Ask:

- How easy is your strategy to explain?
- What other strategies did you try?
- If you were to increase the number of days, would your strategy still work?
- What would you do differently if you were to solve a similar problem?

After the groups have presented their work, illustrate the pattern by saying: "I've noticed that many of your solutions are similar. Many of you recorded the number of toys per day and added up the results. Let's look at the pattern on the board."

Write the following numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. "Which choice would give the winner the most toys? (Solution: The winner received 20 toys in the ten days. It was a better choice for the winner).

Illustrate the pattern by drawing dots on the board in the shape of triangles for the first 5 numbers in the sequence. Ask the students: "What do the shapes look like?"

Explain to students that these numbers are called triangular numbers and were studied by mathematicians in ancient Greece and China. Give students time to extend the pattern to the 20th day. Encourage them to brainstorm the uses of **triangular numbers** in real-life applications. Examples might include the set-up for 10-pin bowling, cheerleaders' pyramids, or displays of soup cans.

Ask the students to complete the second part of the anticipation guide and think about how their thinking about patterns and numbers has changed.

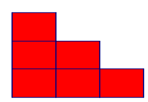
Correct Anticipation Guide responses

1. Agree. Pattern increases by 4, 6, 8, 10.
2. Disagree. Pattern doubles, then increases by 1, so correct response is 15.
3. Disagree. Some do and some don't (for example: 1,2,4,8,16 …)
4. Agree. All patterns, by definition, have rules.
5. Agree.
6. Disagree. There is no rule. (Students may argue that there is a pattern that has not repeated yet. However, no pattern is evident in the sequence given.)

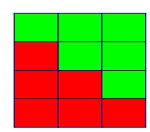
Extensions

Some groups may complete the activity early. Challenge early finishers to determine how many toys the student would receive in 15 days if the same pattern were followed. Encourage students to develop an algebraic pattern rule.

Finding values of triangular numbers



3rd triangular number



rectangle formed from 2 triangles

Have students create a model of the third triangular number. Have them put their model together with a partner's model to form a rectangle. Pose the following questions:

- What is the length of the rectangle? What is the width of the rectangle? What is the area of the rectangle?
- How do the length and width of the rectangle relate to the stage number? Can you use numbers or symbols to describe the pattern?
- Repeat the experiment with the 4th and 5th triangular numbers. Is there a pattern?
- How could you use the pattern to find the 20th triangular number? Any other triangular number?

Teacher note: The algebraic expression for the n^{th} triangular number is: $\frac{n(n+1)}{2}$.

Finding patterns in Pascal's Triangle

Give each student a copy of **PA.BLM6a.2**. Tell the students that they are looking at a partly completed example of Pascal's Triangle, which is a pyramid of numbers. Explain that, while the Chinese investigated properties of the triangle four centuries earlier, it was Blaise Pascal, a French mathematician, who was the first to document this "arithmetical triangle". When presenting Pascal's Triangle to the class, start with the top two rows and ask students to consider how the third row in **PA.BLM6a.2** might be determined from the first two rows. Then ask a similar question about the fourth row. Let students arrive at their own ideas and give them opportunities to explain their reasoning.

Ask:

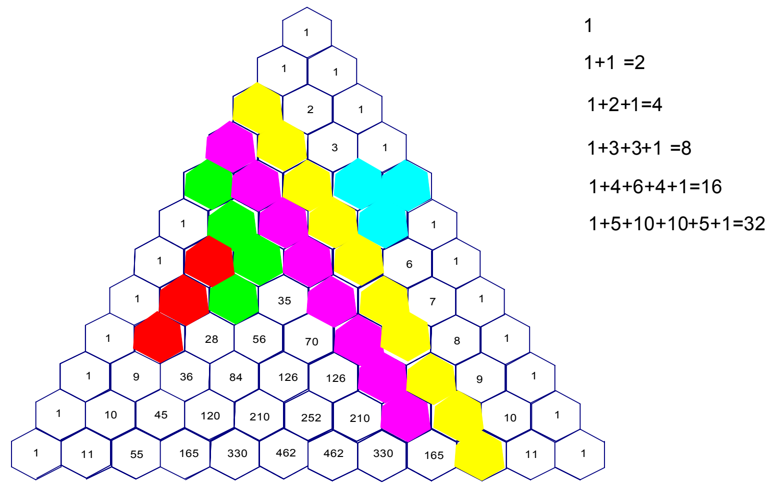
"Can you find any triangular numbers in Pascal's Triangle?"

Highlight examples on the overhead transparency as students point them out. Have students highlight the same examples in yellow on their own copies. Once they have highlighted all the triangular numbers they can find, have them check the accuracy of their findings using a solution transparency.

Next, instruct students to: "Find as many patterns as you can. Indicate each pattern by using a different colour. Create a legend to explain each pattern that you find."

Have students share the patterns they have found, and add the patterns, using different colours, to the image on the overhead transparency.

Some patterns that students might notice in Pascal's Triangle:



- triangular numbers – shown in yellow
- consecutive numbers – shown in red
- two adjacent numbers having the sum of the number below – shown in blue
- a hockey stick pattern (e.g. 1+5+15 = 21) – shown in green (the hockey stick pattern can be any length).
- horizontal sums (shown beside the triangle)
- sums of triangular numbers (e.g., 1 + *3* = 4, 4 + *6* = 10) – shown in purple.

Ask students to solve the problems on the activity cards in **PA.BLM 6b.3: Problems Carousel**. The solutions to the 6 problems are:

1. Person one shakes 6 hands, person two shakes 5 hands, person three shakes 4 hands, and so on…. 6+5+4+3+2+1 is 21. There are 21 handshakes. The sixth triangular number is 21.
2. The 12th triangular number is 78.
3. The 10th triangular number is 55.
4. He could make a stack 13 cans high and would have 9 cans left over. The 13th triangular number is 91 and the 14th is 105.
5. Two consecutive triangular numbers make a square number.
6. Sums of consecutive odd numbers create square numbers: 1+3=4, 1+3+5=9, 1+23+5+7=16

Tiered Instruction

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

Supports for student learning

Understanding the question and developing an appropriate strategy will be benefit students. You may support students by asking:

- What materials could you use in the classroom to model the problem?
- What strategies have you considered for starting to solve the problem?
- How might you organize the information you have been given?

Home Connection

PA.BLM6b.4 extends the problem explored so far by asking, "Which toy was received in the greatest numbers?"

Reviewing the home connection results

When students return the next day, have them work in pairs to share solutions. Encourage pairs to reach a consensus. Ask for students to volunteer strategies and solutions. Record the strategies and solutions on the board and discuss them. In addition, you might also see the following algorithm:

1 chipmunk x 10 days = 10 chipmunks	6 ducklings x 5 days = 30 ducklings
2 blue jays x 9 days = 18 blue jays	7 rabbits x 4 days = 28 rabbits
3 puppies x 8 days = 24 puppies	8 goldfish x 3 days = 24 goldfish
4 kittens x 7 days = 28 kittens	9 ladybugs x 2 days = 18 ladybugs
5 butterflies x 6 days =30 butterflies	10 caterpillars x 1 day = 10 caterpillars

Students may be surprised to learn that there are 2 solutions to the problem.

Assessment

Anticipation Guide

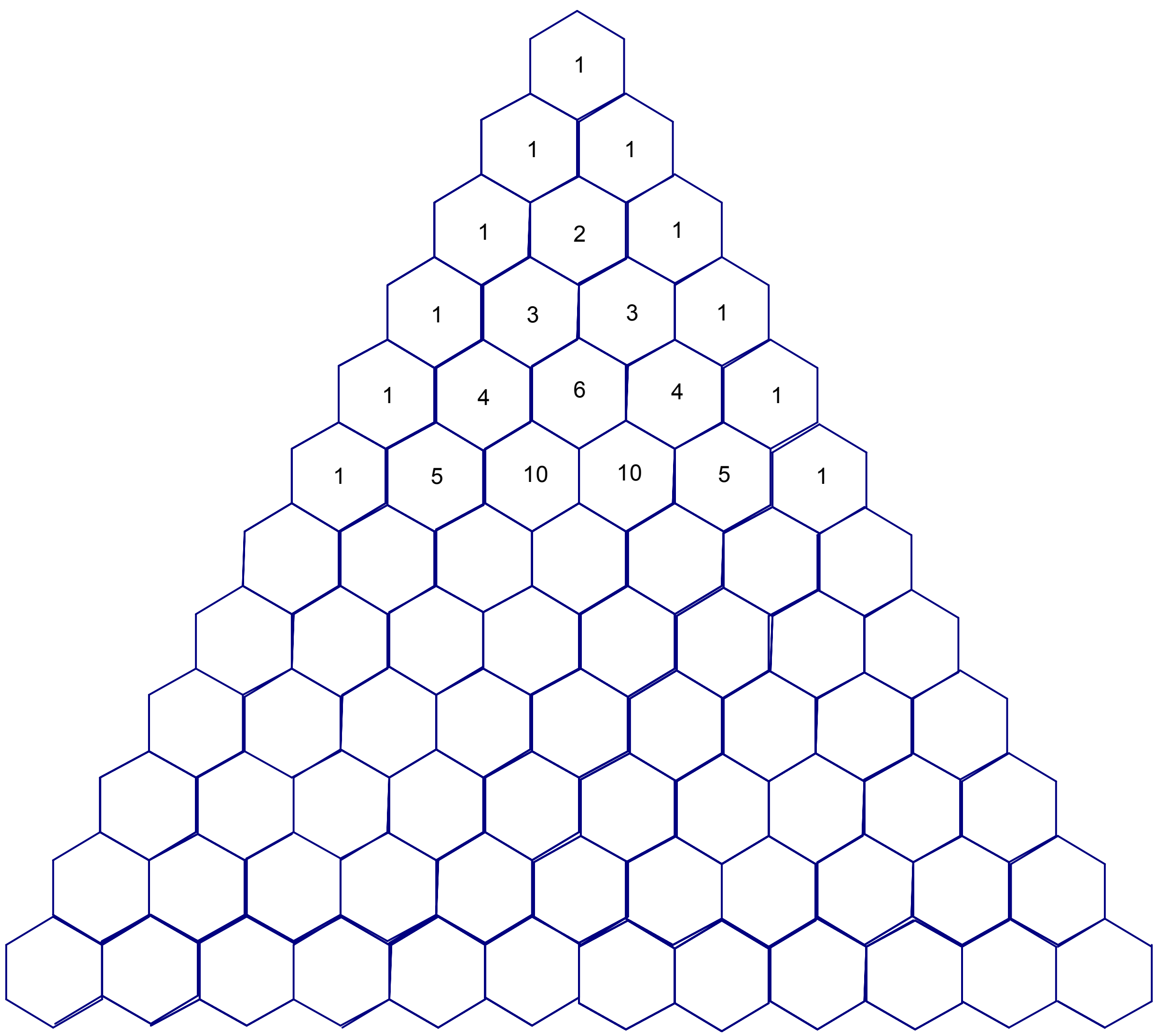
Reconvene the class to discuss the reasons for students' choices on the "after" section of the anticipation guide, and come to a class consensus. Take note of incorrect answers from individual students or large groups. Use this information to determine student understanding and to identify areas of need.

PA.BLM6a.1 Anticipation Guide

Before		**Statement**	**After**	
Agree	Disagree		Agree	Disagree
		1. The next number in the sequence 2, 6, 12, 20… is 30.		
		2. The next number in the sequence 1, 2, 3, 6, 7, 14, … is 28.		
		3. Growing patterns always increase by the same number.		
		4. All patterns have rules.		
		5. You can create patterns by using any of the four operations: +, –, X, ÷.		
		6. The following is a pattern: 3, 7, 10, 22, 24, 30, 29.		

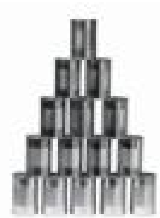
PA.BLM 6a.2 Pascal's Triangle

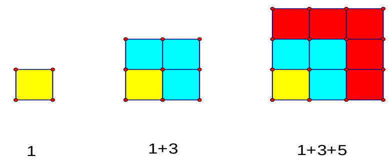
Examine the patterns in Pascal's Triangle. How does Pascal's Triangle grow? How can we use the top two rows to get the third row? How can we use the top three rows to get the fourth row? Fill in the rest of the triangle using the patterns that you notice. Please use pencil.



PA.BLM6a.3 Problems Carousel

Cut out the question cards and set them out at carousel centres. Have “table groups” of students move in a clockwise direction around the room, solving each problem. Provide opportunities for students to display and explain their solutions to the problems. The solutions are under Tiered Instruction: Extensions.

Problem #1: There are seven people in a group. Each person in the group shakes hands with everyone else exactly once. How many handshakes are there? 	**Problem #2:** The clock Big Ben chimes the hour every hour. At 1 o’clock it chimes once, at two o’clock it chimes twice, at three o’clock it chimes three times, and so on. How many times does it chime from 12:01 a.m. to 12:01 p.m.? 
Problem #3: You are making flash cards for a Kindergarten child. You have 10 cards to make. On the first card you write the number 1 and put one sticker on the back. On the second card you write the number 2 and put two stickers on the back. On the third card you write the number 3 and put three stickers on the back. You will continue in this way until you have put ten stickers on the back of the 10th card. How many stickers will you need to make the cards?	**Problem #4:** A grocery store wants to stack cans in a triangular display just like the one in the picture. There are 100 cans to put on display. How many rows high can the cans be stacked? Will there be any cans left over? 

Problem #5: In an experiment Jamal learned that he could make rectangles when he added two of the same triangular numbers together. He wondered what shape he might get if he added together ***consecutive*** triangular numbers. Model the problem and discuss the pattern.	**Problem #6:** Toni noticed a pattern when she added consecutive odd numbers together (starting with 1). What was the pattern? 

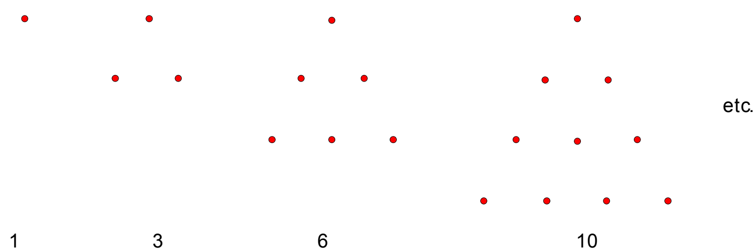
Teacher Note: The cards for each group should be cut out of **PA.BLM6a.1** ahead of time. All but the last card in each set should be paper-clipped together or placed in a sealable plastic baggie. Retain the sixth card for groups that have trouble getting started with the task.

PA.BLM6a.4 **Home Connection**

Beyond Triangular Numbers

Dear Parent/Guardian:

In math we are exploring many types of patterns. Today we focused on triangular numbers. Triangular numbers are numbers that can form a triangle when represented by dots. This is the triangular pattern:



The home activity builds on one we have just completed in class:
At a fun fair, a student wins first prize – a collection of miniature toys. For the next ten days the toys are delivered to the student's classroom in a most unusual way.

On the first day the winner's package contains 1 chipmunk; on the second day the package contains 2 blue jays and 1 chipmunk; on the third day the package contains 3 puppies, 2 blue jays, and 1 chipmunk; on the fourth day the package contains 4 kittens, 3 puppies, 2 blue jays, and 1 chipmunk. The same pattern continues until, on the tenth day, the package contains 10 caterpillars, 9 ladybugs, 8 goldfish, 7 rabbits, 6 ducklings, 5 butterflies, 4 kittens, 3 puppies, 2 blue jays, and 1 chipmunk.

Your child needs to answer these two questions:

1. Which toy do you predict will be received in the greatest numbers during those 10 days?
   Answer: I predict that the student will have received more _______________ than any other miniature toy.
2. Which toy really was delivered in the greatest numbers during those 10 days?

Encourage your child to create a chart or a diagram, or both, to represent his or her solutions. Ask for an explanation of how the answer was found.

Back in class, students will share these solutions with their classmates.

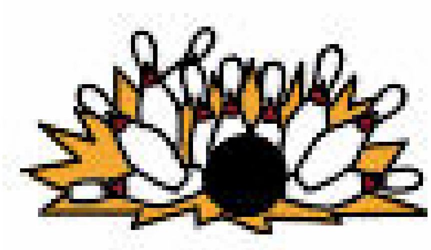
Bowling Dilemma

Strand: Patterning and Algebra, Grade 6

Big Ideas: Expressions, variables, and equations

Overview

In this learning activity students use graphs, tables of values, and equations to compare costs at two different bowling sites, and to determine which site offers the better deal. They also explore the similarities between this problem and one that asks them to compare the top speeds of various animals and to determine which animal would win a race in which the slower animal was given a head start. Students have opportunities to use technology to model the relationships.



Prior to this learning activity students should have had some experience with using variables (symbols or letters), and they should understand the concept of balance in an equation.

Curriculum Expectations

Overall Expectation

- use variables in simple algebraic expressions and equations to describe relationships.

Specific Expectations

- demonstrate an understanding of different ways in which variables are used (e.g., variable as an unknown quantity; variable as a changing quantity);
- identify, through investigation, the quantities in an equation that vary and those that remain constant (e.g., in the formula for the area of a triangle A= $\frac{bxh}{2}$, the number 2 is a constant, whereas b and h can vary and may change the value of A);

- solve problems that use two or three symbols or letters as variables to represent different unknown quantities;
- determine the solution to a simple equation with one variable, through investigation using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (***Sample problem***: Use the method of your choice to determine the value of the variable in the equation 2 x N + 3 = 11. Is there more than one possible solution? Explain your reasoning.).

About the Learning Activity

Time: 90 minutes

Materials

- graph paper
- **PA.BLM6b.1: Bowling Dilemma**
- **PA.BLM6b.2: Home Connection**

Math Language

- variable, constant, equation, intersection

Instructional Grouping: "table groups" (4-6 students) and pairs

About the Math

Variable: A quantity, represented by a letter or symbol, that can have one or more values. For example, in P = 4s (the formula for the perimeter of a square), P and s are variables and 4 is a constant. Variables can be placeholders for missing numbers (as in □ + □ = 14 and 2x = 8), or they can represent quantities that vary (as in P = 4s and y = 2x + 1).

Constant: A quantity that stays the same.

Algebraic Equation: A mathematical sentence that contains variables, constants, and an equal sign (e.g., 4x = 12, x + y = 15). **Note:** Solving an equation means finding values for the variable(s) that make the equation true.

Getting Started – Warm Up (Part 1 of a 3 part lesson)

The speed of animals

Present the following facts to students.

"Predict which animal runs the fastest:

- Pronghorn antelope
- Garden snail
- House mouse
- Reindeer
- Giant tortoise
- Kangaroo

Teacher note: For the top speeds of various animals, do an Internet search using the key words "animal speeds". Here are the (approximate) top speeds of the animals in this problem:

- pronghorn antelope: 28 m/s
- kangaroo: 14 m/s
- reindeer: 4 m/s
- house mouse: 2 m/s
- giant tortoise: 3 m/min.
- garden snail: 0.5 m/min.

Allow students time to share their prediction with their elbow partner. Reveal the relationships between the running speeds of the animals and ask students to compare it to their prediction. "A pronghorn antelope can run twice as fast as a kangaroo. A reindeer can run four times as fast as a house mouse. A giant tortoise can move six times as fast as a garden snail." Ask: "Suppose that an antelope and kangaroo had a race, and the kangaroo had a head start of 100 m. Who would win the race?"

Ask students to work in pairs on the problem, then discuss solution ideas as a whole class. Draw attention to the various methods that students use to analyse the problem, such as drawing a diagram, making a T-chart, or acting out the situation. Students will notice that the length of the race has not been specified. Ask them to consider different race lengths to see if length makes a difference.

Working on It – (Part 2 of a 3 part lesson)

Bowling Dilemma

Distribute **PA.BLM6b.1** to students and do a shared reading of the scenario with the class:

You want to invite your friends to a bowling competition. You haven't yet decided how many games will be played in the competition, but you are confident that you will play at least 2 games and at most 5 games. There are two bowling sites quite near by, and you are trying to decide which site offers the better deal. At site #1 each person would pay a rate of $3.50 per game plus a one-time $3.00 fee for shoe rental. At site #2 each person would pay a rate of $4.50 per game, but there is no shoe rental fee. Which site is the better value?

Creating algebraic equations to solve the problem

Using a think aloud method, pose the following statements to the class before the students begin working on the problem:

"I think":

- I need an equation for the cost of playing bowling at site #2.
- The cost itself is one of the variables. I'll call this variable C. My equation so far is: C = …

"I wonder":

- What other variables are there? What letters could I use to represent them?
- What constants are there?
- How can I write the equation?
- How I will test the equation?

Have students work in pairs to solve the problem. After students have worked for a while and have developed some methods of representing the different sites, create a chart and record the students' ideas on how to represent each site.

Site #1

C = 3.50 x G + 3

- C represents the cost
- $3.50 is the cost per game
- G represents the number of games that will be played
- $3.00 is the equipment rental fee

Site #2

C = 4.50 x G

- C represents the cost
- $4.50 is the cost per game
- G represents the number of games that will be played

Once most pairs of students have developed their second equation, discuss the equation as a whole class, retracing the steps modelled above.

Scaffolding suggestion: Provide the headings for each column.

Creating a table of values

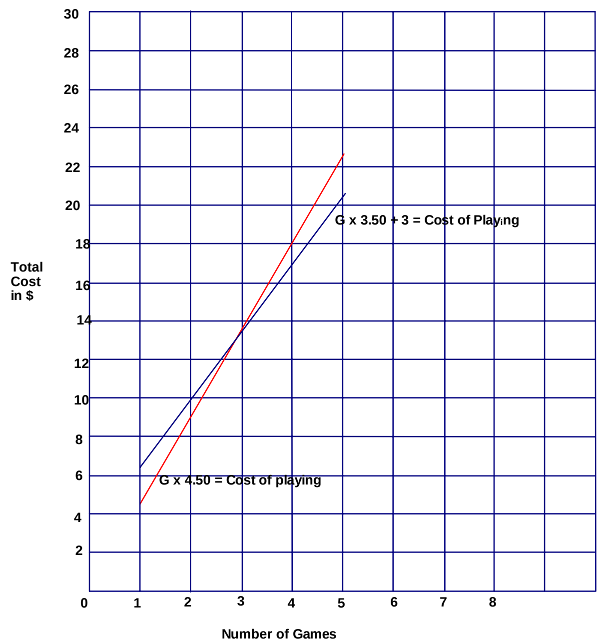
Explain to students that they will now be calculating the costs at each site using the equations they have just developed. With student assistance, model how to use the equations to calculate the cost of one game at each site, and enter the data in a table of values such as the one at right. Complete a row as a whole class, and then send students back to their small groups or pairs to continue their work. Ask them to include a table of values in their solutions.

Number of Games	Site #1 ($)	Site #2 ($)
1	3.50(1)+3 = 6.50	4.50(1) = 4.50
2		
3		
4		
5		

Looking Back

Review students' solutions and select two or three to show how students used the table of values. Explain that another way to represent this information is by using a graph. See **PA.BLM6b.1**.

Instruct students to plot both costs on the same graph so that they can see where the costs intersect. Students could use graph paper or chart grid paper, or spreadsheet software to create line graphs based on the data in the table. Ask students to compare their table of values and their graph to see if the talbe and graph support their answer to the original problem. Ask them to explain their thinking to another group. (The table and the graph below compare the costs of playing up to 5 games. Site #2 offers the better deal until it intersects with site #1 at the 3rd game. After this, site #1 becomes the better choice. The cost is the same for both sites at the point where the two graph lines intersect.)



Number of Games	Site #1 ($)	Site #2 ($)
1	6.50	4.50
2	10.00	9.00
3	13.50	13.50
4	17.00	18.00
5	20.50	22.50

Reflecting and Connecting – (Part 3 of the 3 part lesson)

Facilitate a class discussion of the bowling problem. Ask:

- Is there just one solution?
- Which site would you pick? Why?
- What does the intersection point on the graph represent?

Extensions

Revisiting the Speed of Animals problem

Present to students the (approximate) top speeds of the animals used in the Speed of Animals problem:

- pronghorn antelope: 28 m/s
- kangaroo: 14 m/s
- reindeer: 4 m/s
- house mouse: 2 m/s
- giant tortoise: 3 m/min.
- garden snail: 0.5 m/min.

Pose the following problem:

"Suppose that an antelope and kangaroo had a race, and the kangaroo had a head start of 100 m. Who would win the race?"

Ask them to work in pairs to do the following:

- Create algebraic equations to model the speed of each animal.
- Create a chart showing the distance travelled by each animal over a period of 10 seconds.
- Identify the constants and variables in the expressions.

Contrast the equations and the graphs for the two problems. What similarities do you notice? What differences are there?

Facilitate a class discussion of the ideas that emerge. See the Teacher Note above for some ideas that students might share about the equations. They will also notice that both sets of graphs are similar in that they have two lines that intersect and the point of intersection helps us to solve each problem.

Tiered Instruction

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

Supports for student learning

- **Bowling dilemma.** Give students separate charts to generate ordered pairs.
- **Equations.** Use different colours for the variable(s) and the constant.

Extensions

Bowling dilemma extension. Challenge students to create an equation for a third site, whose costs will fall between the costs of the other two.

Home Connection

See **PA.BLM6b.3**.

Assessment

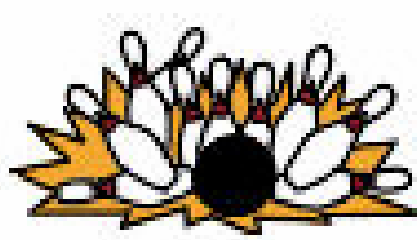
Assessment opportunities might include:

- paragraphs in a reflective journal, completed at school, which explain the Home Connection poster activity;
- informal discussions with students during the activity.

PA.BLM6 b.1 Bowling Dilemma

You want to invite your friends to a bowling competition. You haven't yet decided how many games will be played in the competition, but you are confident that you will play at least 2 games and at most 5 games. There are two bowling sites quite near by, and you have to decide which site offers the better deal. At site #1 each person would pay a rate of $3.50 per game plus a one-time $3.00 fee for shoe rental. At site #2 each person would pay a rate of $4.50 per game but there is no shoe rental fee. Which site is the better value?

- Create algebraic equations to model the cost at each site.
- Create a chart showing the cost of the first 5 games for both sites.
- Identify the constants and variables in the expressions.
- Plot both sets of data on a line graph. Use a different colour for each line. Where do the lines intersect (cross)? Explain what the graph shows.
- Which site is the better value? What assumptions have you made?
- If you played 20 games at site #1, how much money would you have to pay? Explain your reasoning.



PA.BLM6b.2 **Home Connection**

Constants and Variables

Dear Parent/Guardian:

In class we have been studying constants and variables in equations. We have explored relationships between constants and variables and have shown them in the form of charts, graphs, and algebraic equations.

Please ask your child to create a poster using numbers, pictures, and words to illustrate the concept of constants and variables, For example: