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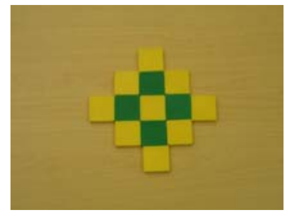
Patterning and Algebra Learning Activities – Grade 5

Growing Weave Designs

Strand: Patterning and Algebra, Grade 5

Big Ideas: Patterns and relationships

Overview



In this activity, students investigate growing patterns by modelling the growth of a design, using colour tiles or interlocking cubes. They represent growth with concrete materials and they generate patterns (such as 4, 8, 12, 16… and 1, 4, 9, 16, 25…). They record patterns and generalize pattern rules using words to describe relationships. They represent the patterns in different ways, using tables, diagrams, graphs, and manipulatives. The context of the activity uses a Grade 5 art project in which students have woven different coloured papers to create a design. The teacher wonders if the students are able to recognize and represent the math found within their weave designs. Prior to this learning activity, students should have had some experience with representing patterns using charts, diagrams, graphs, and concrete materials.

Curriculum Expectations

Overall Expectation

- determine, through investigation using a table of values, relationships in growing and shrinking patterns, and investigate repeating patterns involving translations.

Specific Expectations

- create, identify, and extend numeric and geometric patterns, using a variety of tools (e.g., concrete materials, paper and pencil, calculators, spreadsheets);
- build a model to represent a number pattern presented in a table of values that shows the term number and the term;
- make a table of values for a pattern that is generated by adding or subtracting a number (i.e., a constant) to get the next term, or by multiplying or dividing by a constant to get the next term, given either the list of terms (e.g., 12, 17, 22,

27,32,…) or the pattern rule in words (e.g., start with 12 and add 5 to each term to get the next term);

- make predictions related to growing and shrinking geometric and numeric patterns.

About the Learning Activity

Time: 120 minutes

Materials

- **PA.BLM5a.1: The Weave Design Growth Pattern**
- **PA.BLM5a.2: A Variation on the Weave Design Growth Pattern**
- **PA.BLM5a.3: Graphing Design Growth Patterns**
- **PA.BLM5a.4: Home Connection: Square Number Investigation**
- 100 coloured tiles or interlocking cubes (50 of each colour) for each pair of students
- graph paper

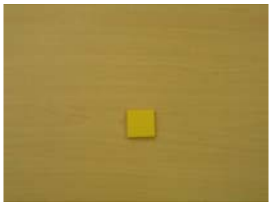
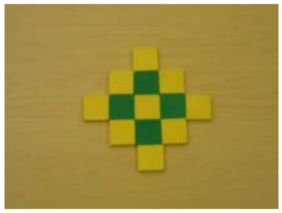
Math Language

- growth pattern, array, T-chart, square numbers, diagonal

Instructional Grouping: pairs

About the Math

The design growth pattern

We can show the growth pattern by using tiles of two different colours. In the first image, a yellow tile represents the design at its starting point. In the second image, green tiles show the growth of the design that has resulted. In the third image, the green tiles are now completely surrounded by yellow tiles, representing the next growth stage. The pattern alternates, with each colour surrounding the shape in turn. When modelling the pattern with tiles, you do not need to start over again to show each growth stage – just continue building onto the previous stage. The crystal grows in a number of interesting ways:

- The pattern generated by the number of Colour 1 tiles is 1, 1, 9, 9, 25, 25 … . This is a repeating and growing pattern that can be made by squaring the odd numbers and repeating each term twice.
- The Colour 2 tile pattern is represented by the square of even numbers.
- The additional tiles needed to complete each stage are represented by the multiples of 4 (4, 8, 12, 16 …).
- Counting tiles in the rows (or columns) gives the pattern 1 + 3 + 1 for stage 2, 1 + 3 + 5 + 3 + 1 for stage 3, and so on.

- The rule for successive stages is: the stage number squared plus the stage number minus 1 squared will give the number of total tiles.

Stage	Colour 1	Colour 2	Total
1	1 1 x 1 = 1	0	1
2	1 1 x 1 = 1	4 2 x 2 = 4	5 1 + 4 = 5
3	9 3 x 3 = 9	4 2 x 2 = 4	13 5 + 8 = 13
4	9 3 x 3 = 9	16 4 x 4 = 16	25 13 + 12 = 25
5	25 5 x 5 = 25	16 4 x 4 = 16	41 25 + 16 = 41
6	25 5 x 5 = 25	36 6 x 6 = 36	61 41 + 20 = 61
7	49 7 x 7 = 49	36 6 x 6 = 36	85 61 + 24 = 85
8			
9			
10	81 9 x 9 = 81	100 10 x 10 = 100	181 145 + 36 = 181

Square numbers

Square numbers are formed by multiplying a number by itself. The first seven square numbers are examined in this activity: 1, 4, 9, 16, 25, 36, 49. Square numbers get their name from the fact that they form a square when put in an array. The square numbers can be expressed using multiplication or exponents: $1 = 1 \times 1 = 1^2$, $4 = 2 \times 2 = 2^2$, $9 = 3 \times 3 = 3^2$, $16 = 4 \times 4 = 4^2$. The meaning of such exponents can be linked to area measurement units, such as cm^2 and m^2.

*	* *	* * *	* * * *
	* *	* * *	* * * *
		* * *	* * * *
			* * * *
1	4	9	16

Although the math above stretches beyond the Grade 5 level, understanding the richness of the patterns in this design provides the teacher with deep knowledge of the possibilities inherent in the design. Seeing the possibilities in the design enriches the teacher's ability to recognize important math learning in the students' work and dialogue which will become the underpinnings of future math study.

Getting Started – Warm Up (Part 1 of a 3 part lesson)

Equal signs

Students often have a fragile understanding of the meaning of the equal sign. Begin the lesson with a series of equations. Write each equation one at a time on the board and ask the students to decide what the missing number is in the box. The box activity offers an opportunity to discuss variables as a representation of unknown quantities.

$$5 + 7 = \square$$
$$5 + \square = 12$$
$$\square + 7 = 12$$
$$\square + \square = 12$$
$$5 + \square = \square$$

As students work through the solutions, ask them to represent or prove their answers using concrete materials. Many students misunderstand that the equal sign is a symbol between two equal values and, instead, think it means the answer is next (sum, product, quotient) regardless of where the variable occurs. On the last two equations list possible solutions that the students provide and ask students to look for patterns in the results:

□ +	□ = 12	5 + □	= □
1	11	1	6
2	10	2	7
3	9	3	8
4	8	4	9
5	9	5	10

Students benefit from ongoing opportunities to explore equations with different operations using variables on different sides of the equation.

The Growing Design

Distribute **PA.BLM5a.1** and have students work with their elbow partner to build the first three stages of the design as pictured on the **PA.BLM5a.1** (see also below), and ask them to record the information on the chart. Circulate among the students and ensure that they are completely surrounding the design on each stage and are correctly recording the information. As the pairs of students build each stage, ask:

- How might you describe the pattern to someone who can't see it?
- How many tiles of each colour are you adding?
- Predict how many tiles you'll need for stage 4.
- Did you notice how much the total number of tiles increases from stage to stage?
- Do you think you have enough tiles to complete stage 7?

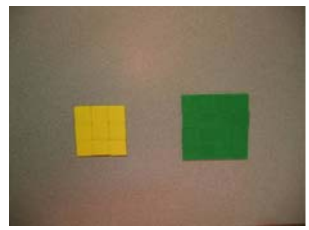
> **Teacher Note:** Constructing the design using manipulatives is critical for several reasons. Students link patterns to spatial thinking and geometry. Visual representations help to develop abstract thinking. Furthermore, students require a concrete representation in order to make sense of the pattern.

Working on It – (Part 2 of the 3 part lesson)

Exploring growth patterns

Have students continue to work in pairs to complete stages 4 through 7, recording how many tiles of each colour they used and totalling the number of tiles used at each stage. Prompt students to discuss what they are doing and what patterns they see as they build successive stages (see the About the Math section). They will need to use the patterns they discover in order to extend the pattern to stage 10. Ask:

Visual scaffolding. Have the student complete stage 4. Ask the student to decompose the tiles into two piles, according to colour, and then to arrange each colour of tiles into an array. There should be 9 tiles of one colour and 16 of another, both arrays forming squares. Repeat for stages 5 and 6.

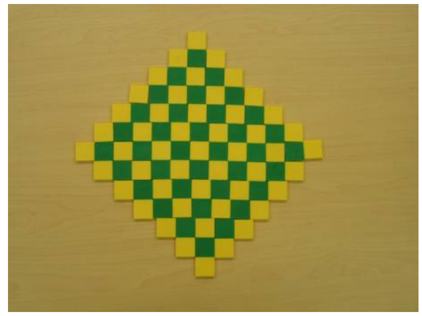


- "What is the pattern in the number of additional tiles needed to create each stage?"
- "What is the relationship between the stage number and the number of one of the colours of tiles added?"
- "How many tiles do you predict will be needed for stage 10?"

Some students may notice that the number of tiles on the side of the square is the stage number (the number of tiles in the square is the stage number multiplied by itself – stage number x stage number). This is a square number and can be expressed as the stage number squared.

Other students might look at the overall pattern as the number of tiles increases. Encourage these students to find the difference between the totals in order to discover the number of tiles added each time. When recording the differences they may notice that the pattern increases by multiples of 4 (4 is added first, then 8, then 12 then 16, and so on).

Shown on the right is the design completed to the end of stage 7. There are 49 yellow tiles and 36 green tiles for a total of 85 tiles. Some students may notice that there are 7 yellow tiles along one edge and that this is the 7th stage, for a total of 7 x 7 or 49 yellow tiles.



Remind students that they need to draw on graph paper a diagram of the growth of each stage, using coloured pencils or markers.

Reflecting and Connecting – (Part 3 of a 3 Part Lesson)

When students have completed predicting the number of tiles needed for stage 10, initiate a class discussion to share different pattern rules. Ask students to check that each suggested rule actually works. Make a chart of rules that were used to solve the problem and post it in the class. See the About the Math section for a list of possible rules.

Teacher note: The students' suggested rules may not always be the most efficient. However, students need to "work to make sense of the problem in their own way. They (should) look for patterns and for connections with other problems." (*Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004*). After this exploration stage, you can then help students move towards more efficient methods.

Tiered Instruction

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

Supports for student learning

- Some students may need to use a calculator to help them discover patterns such as the difference or sum of different colours of tiles.
- Some students may complete the growth sequence but not identify the pattern of coloured tiles. They would benefit from deconstructing the pattern and assembling the tiles into arrays.

Extensions

- **The 20th stage.** Ask students who finish early to use their rule to find the number of tiles in the 20th stage without having to extend the chart. Solving this new problem will help to deepen their understanding of the pattern.
- **A variation on the design growth pattern.** A variation of the activity can be found in **PA.BLM5a.2**. Provide students with tiles of a third colour and ask them to add tiles in this colour to the existing design patterns to create squares. Ask them to model the first three stages, just as they did in the original design growth activity. They should record their results in the table of **PA.BLM5a.2**, then look for patterns and develop rules for sequences. A completed table is shown below. Students may find the following patterns:

  - The number of Colour 3 tiles is a multiple of 4 (0, 4, 12, 24…).
  - The total area is always the square of an odd number.
  - The perimeter begins at 4 and increases by 8 for each stage (4, 12, 20, 28…)
  - The total area is given by (the number of Colour 3 tiles used) x 2 + 1. For example, for stage 3, total area (12 Colour 3 tiles) x 2 + 1 = 25

Stage	Number of Colour 3 Tiles used	Total Area	Perimeter
1	0	1	4
2	4	9	12
3	12	25	20
4	24	49	28
5	40	81	36
10	180	361	76

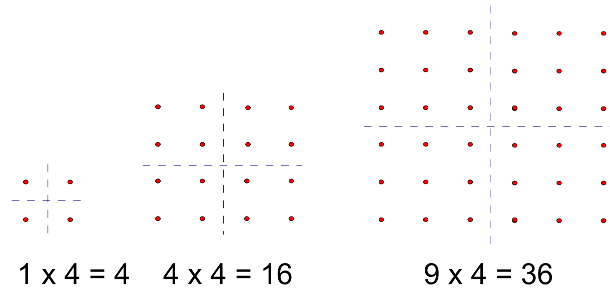
- **Graphing the patterns.** For this extension, students will need a completed **PA.BLM5a.1**, **PA.BLM5a.3**, and a calculator to help them identify pattern rules represented by graphs. Ask them to choose from a variety of given graphs and to match the appropriate graphs to the patterns discovered for Colour 1, Colour 2, and the Total Area patterns (**PA.BLM5a.1**). Ask students to justify their choices.

- Solutions: Colour 1 – Graph A; Colour 2 – Graph F; Total Area – Graph D. Challenge students who finish early to work out a pattern rule for one of the incorrect graphs. Only Graph B has a pattern rule. The pattern increases as follows: 1, 3, 6, 10, 15, 21. The terms are increased by adding consecutive numbers; for example: 1 + 2 = 3, 1 + 2 + 3= 6, 1 + 2 + 3 + 4 = 10.

Home Connection

See **PA.BLM5a.4: Home Connection – Investigating Square Numbers**.

Reviewing the home connection results. Students should have noticed that multiplying a square number by another square number will always result in a square. To reinforce this point, reproduce on the board the diagram on the right. Explain that the rule can also be seen by looking at the factors for the numbers involved. For example, 9 x 4 can be written as 3x3 x 2x2. To be able to create a square number, we have to be able to regroup the factors so that they result in the product of two identical numbers, such as: 3x3 x 2x2 = 3x2 x 3x2 = 6x6. This type of result is never possible when multiplying a square number by a non-square number. For example, 9 x 6 can be expressed as 3x3 x 3x2. We cannot regroup these factors to form a product of two identical numbers.



Assessment

The completed home connection task can be used to assess the student's thinking process. A brief conversation with each child will reveal his or her level of understanding. The written paragraphs requested in **PA.BLM5a.2** and **PA.BLM5a.4** can be used to judge the student's communication skills and ability to use appropriate mathematical language.

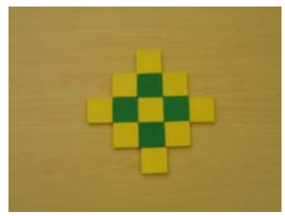
PA.BLM5a.1 The Design Growth Pattern

An art teacher asked Grade 5 students to explore different ways to cut and weave paper together. When they showed their weaves to their homeroom teacher, the teacher noticed patterns in their work. He wondered if the students could represent the patterns in their weaves with tiles.

Your task is to model the growth of the design, using tiles or interlocking cubes, and then to record your results in a diagram and chart. You will need 2 different colours of tiles or cubes. The first three stages of growth are shown below.


Stage 1


Stage 2


Stage 3

Use your tiles or cubes to extend the growth pattern to stage 7, and record your results in the chart. Draw a diagram on graph paper to show the growth.

Stage	Number of Colour 1 Tiles Used	Number of Colour 2 Tiles Used	Total Number of Tiles Used
1			
2			
3			
4			
5			
6			
7			

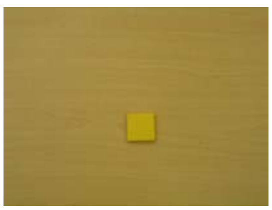
What different patterns can you see? How can you use those patterns to help you predict:

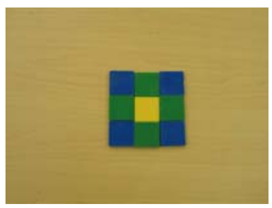
- the number of Colour 1 tiles at stage 10?
- the number of Colour 2 tiles at stage 10?
- the total number of tiles at stage 10?

PA.BLM5a.2 A variation on the design growth pattern

Here, you will use the design patterns generated in the previous activity to extend your understanding of patterns and to explore the concept of area.

Begin by adding another set of colour tiles to the existing crystal patterns to create squares. You may use the diagrams you created in the previous activity to help you.





After you have modelled the stages, complete the following chart. Record your results, look for patterns, and develop rules for the patterns.

Stage	Number of Colour 3 tiles used	Total area	Perimeter
1			
2			
3			
4			
5			

Develop pattern rules for:

- the number of Colour 3 tiles used;
- the total area;
- the perimeter.

Are there any relationships between the columns?

Using the pattern rules you've discovered, determine the number of Colour 3 tiles, the total area, and the perimeter for stage 10. Think about how you could do this without extending the chart.

Write a brief paragraph explaining how you found the area of each of the squares at each stage.

PA.BLM5a.3 Graphing the design patterns

Match the graphs that represent the data from the Design Pattern activity (**PA.BLM5a.1**).

The graph that represents the growth pattern for Colour 1 is: ______________

I believe that I am right because: ________________________________

____

The graph that represents the growth pattern for Colour 2 is: ______________

I believe that I am right because: ________________________________

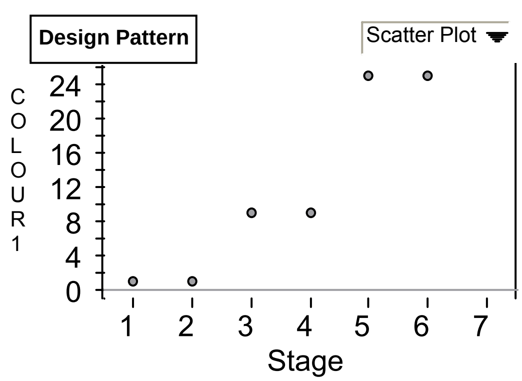
____

The graph that represents the growth pattern for the Total is: ______________

I believe that I am right because: ________________________________

____

A



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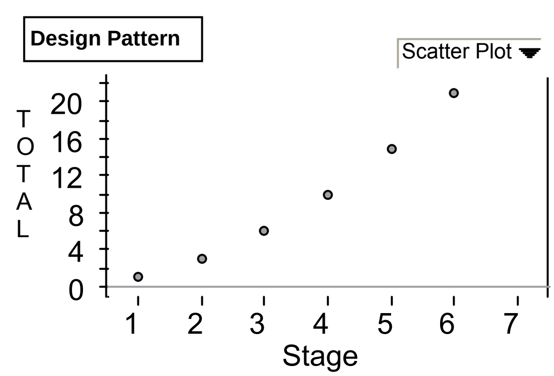

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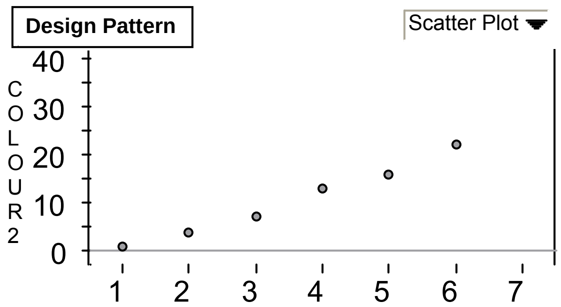
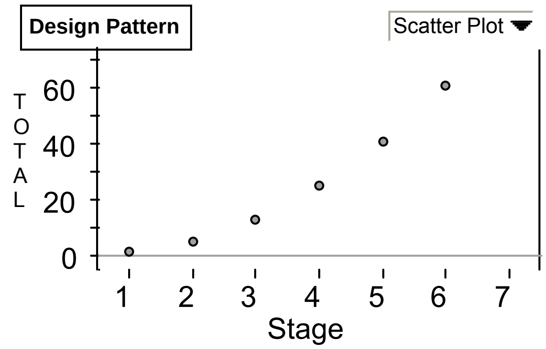
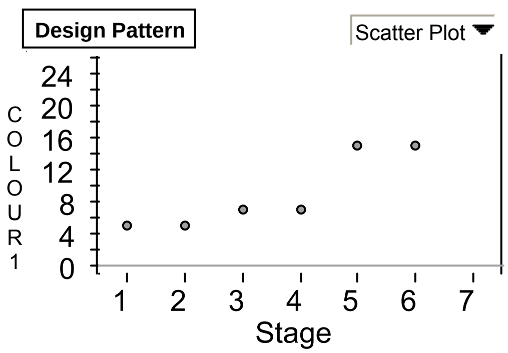
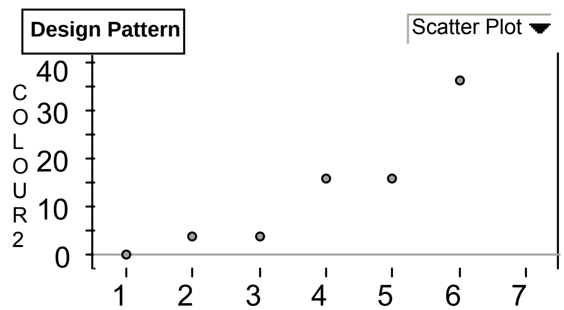

Design Pattern
Scatter Plot
COLOUR 2
40
30
20
10
0
1 2 3 4 5 6 7

D

Design Pattern
Scatter Plot
TOTAL
60
40
20
0
1 2 3 4 5 6 7
Stage

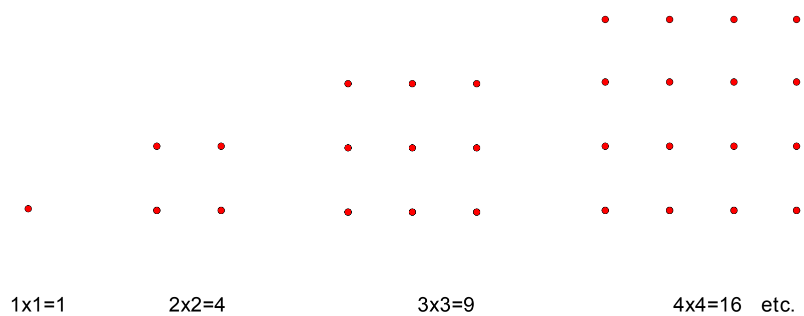
E

Design Pattern
Scatter Plot
COLOUR 1
24
20
16
12
8
4
0
1 2 3 4 5 6 7
Stage

F

Design Pattern
Scatter Plot
COLOUR 2
40
30
20
10
0
1 2 3 4 5 6 7

PA.BLM 5a.4 Home Connection
Square Number Investigation

Dear Parent/Guardian:

In math we are exploring many types of patterns. Square numbers are formed by multiplying a number by itself. They are called square numbers because they can form a square when drawn as an array of dots (see below).
The square number pattern is:



This home connection activity builds on the work we have just completed in class. Please ask your child to demonstrate his or her understanding by writing a paragraph on each of the statements below. The paragraphs may include diagrams and/or numbers to help illustrate the reasoning behind the responses.

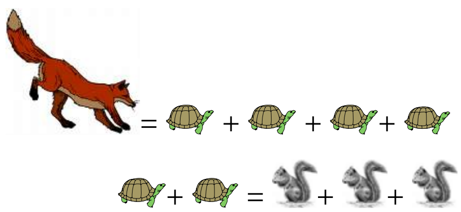
- If a square number is multiplied by another square number, the result is always a square number. Do you agree or disagree? Explain your thinking.
- If a square number is multiplied by a non-square number, the result is sometimes a square number. Do you agree or disagree? Explain your thinking.

In class, students will be sharing their paragraphs with their classmates.

Balancing Act

Strand: Patterning and Algebra, Grade 5

Big Ideas: Expressions, variables, and equations



Overview

In this learning activity students explore the concept of equality and variables. In the context of a literature connection, students use symbols and letters to represent the masses of different animals and to investigate relationships and express them algebraically. Students identify relationships between the masses of the animals and write equations that represent balanced tug-of-war teams. This lesson will employ the Bansho technique of interpreting and analysing peer solutions to deepen understanding. In a Bansho, each group of students shows their work to the class. The solutions are hung on the board for all the class to see, and comparisons are made between solutions. Similar solutions are hung together, and the samples are annotated by the teacher. The teacher may choose to organize the student samples by strategy, number of solutions, chose representations, or communication. The Bansho is not used to level the work, but to analyse the work to make math concepts explicit. Prior to this learning activity, students should have had some experience in using variables (symbols or letters) in expressions, and in modelling with concrete materials.

Curriculum Expectations

Overall Expectation

- demonstrate, through investigation, an understanding of the use of variables in equations.

Specific Expectations:

- demonstrate, through investigation, an understanding of variables as changing quantities, given equations with letters or other symbols that describe relationships involving simple rates (e.g., the equations C = 3 x n and 3 x n = C both represent the relationship between the total cost (C), in dollars and the number of sandwiches purchased (n), when each sandwich costs $3);
- demonstrate, through investigation, an understanding of variables as unknown quantities represented by a letter or other symbol (e.g., 12 = 5 + □ or 12 = 5 + s can be used to represent the following situation: "I have 12 stamps altogether and 5 of them are from Canada. How many are from other countries?");
- determine the missing number in equations involving addition, subtraction, multiplication, or division and one- or two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator).

About the Learning Activity

Time: 120 minutes

Materials

- **PA.BLM 5a.1: Fair Tug-of-War Teams**
- **PA.BLM 5b.2: Wolf Extension Activity**
- **PA.BLM 5b.3: Home Connection: A Massive Puzzle**
- Chart paper and markers
- manipulatives such as base ten blocks, interlocking cubes, coloured tiles

Math Language

- equation, variable, identity, infinite

Instructional Grouping: whole class, pairs

About the Math

Variable: A quantity, represented by a letter or symbol, that can take on one or more values. For example, in P = 4s (the formula for the perimeter of a square), P and s are variables and 4 is a constant. Variables can be placeholders for missing numbers (as in □ + □ = 14 and 2 x = 8) or they can represent quantities that vary (as in P = 4s and y = 2x + 1).

Algebraic Equation: A mathematical sentence that contains a variable or variables and an equal sign (e.g., 4x □ = 12, x + y = 15). Solving an equation means finding values for the variable that make the equation true.

> **Teacher note:** When working with equations, it is important to put variables on both sides of some of the equations, so that students will come to recognize that equations are a balancing across the equal sign of both numbers and variables.

Getting Started – Warm Up (Part 1 of a 3 part lesson)

Equal signs

Students often have a fragile understanding of the meaning of the equal sign. Begin the lesson with a series of equations. Write each equation one at a time on the board and ask the students to decide what the missing number is in the box. The box activity offers an opportunity to discuss variables as a representation of unknown quantities.

5 + 7 = □

5 + □ = 12

□ + 7 = 12

□ + □ = 12

5 + □ = □

As students work through the solutions, ask students to represent or prove their answers using concrete materials. Many students misunderstand that the equal sign is a symbol between two equal values and, instead, think it means the answer is next (sum, product, quotient) regardless of where the variable occurs. On the last two equations list possible solutions that the students provide and ask students to look for patterns in the results:

□ +	□ = 12	5 + □ =	□
1	11	1	6
2	10	2	7
3	9	3	8
4	8	4	9
5	9	5	10

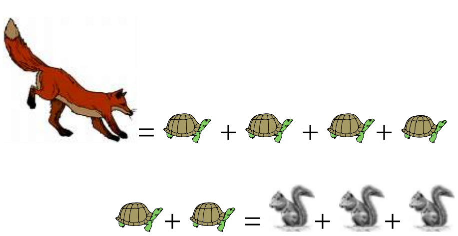
Students benefit from ongoing opportunities to explore equations with different operations using variables on different sides of the equation.

Working on It - (Part 2 of a 3 part lesson)

Read the book *Equal Shmequal*, by Virginia Kroll. Pause to discuss with students the mathematics that emerges. Use letters to represent the animals in the story and, with help from the students, write equations to represent the balancing of their masses. If you do not have access to the book, read the following premise to the class:

Teeter-totter note: Students will likely know that the balance can be thrown off if you sit closer to the centre (fulcrum) of the teeter-totter. For the animals to balance their masses accurately they will need to be standing on top of one another, as depicted in **PA.BLM5b.2**.

"A little brown squirrel sits on a branch of a tree in a park, watching children play. He hears one of the children say that teams have to be equal for the tug-of-war game they are about to play. The children are called away, leaving their rope behind. The squirrel is joined by a fox, four turtles, and two more squirrels. They decide to use the rope to play a tug-of war game. The first squirrel insists that the teams have to be equal. They decide that the teams should have equal mass. They use a teeter-totter as a balance scale to test their weights. After various trials, they manage to make the teeter-totter balance. Using the teeter-totter, they discover the following relationships between their masses:



Teacher note: Although mass is considered directly proportional to strength for the purposes of this activity, such a relationship is not always the case in real life.

The two balanced teams then compete in a very even contest that neither side can win."

Writing the equations

Tell students to work with their elbow partner to draw a diagram or an equation showing how the masses of the animals in the story are balanced. Ask the class to brainstorm ways they could represent each animal without having to draw each animal. Students may suggest using letters, shapes, or graphics. Student anticipated response may include:

> **Scaffolding suggestion:** To model the relationship, students may use manipulatives such as paper cut-outs of the animals, or the letters F, T, and S written on square tiles.

F for fox, T for turtle, and S for squirrel.

F = T + T + T + T

T + T = S + S + S

OR

□ = Fox ● = Turtle ▲ = Squirrel

□ = ●+●+●+●

●+● = ▲+▲+▲

Students will select symbols or variables that make sense to them.

Pose the problem for students:
How many different ways can the animals combine on either side of the teeter totter to make the teeter totter balance if you have a fox, 4 turtles, and 6 squirrels?

Understanding the Problem

Ask the students to turn to a classmate and explain what they think the problem is asking them to do. Ask students to place their hand on the top of their head if they feel sure they understand the problem, and ask students who are still wondering to put their fist under their chin as if they are still thinking (hmmm). Observe the class to see if students feel they fully understand the problem. Ask a student with hand on head to explain the problem. Ask a student who was not sure to reiterate what the first student said and to ask any questions for clarification. Survey the class again; organize students into groups of 3 or 4 once the class understands the problem.

Make a Plan: Creating new equations

Observe the groups as they begin their work. Ask students at each group to articulate their strategy for creating their balance equations. Use a sticky note to record the strategies students are able to articulate, and also note those strategies that students are employing but are not able to describe. If some groups have trouble beginning, ask a few groups to share the strategy they have chosen with the whole class.

Carry Out the Plan

As students begin to record their equations, ask students to explain their solutions. Resist the urge to direct students to specific equations. Deeper learning can be achieved by asking questions such as:

How is this side of the equal sign balanced with the other side?

I'm still not sure I understand your thinking, can you think of another way to show me?

I see you have drawn manipulatives to show your equations; is there another way to record your thinking?

How do you know the equation is true?

Continue to observe the students as they work, and call the group back together when you feel the groups have exhausted the solutions they are able to create.

Looking Back

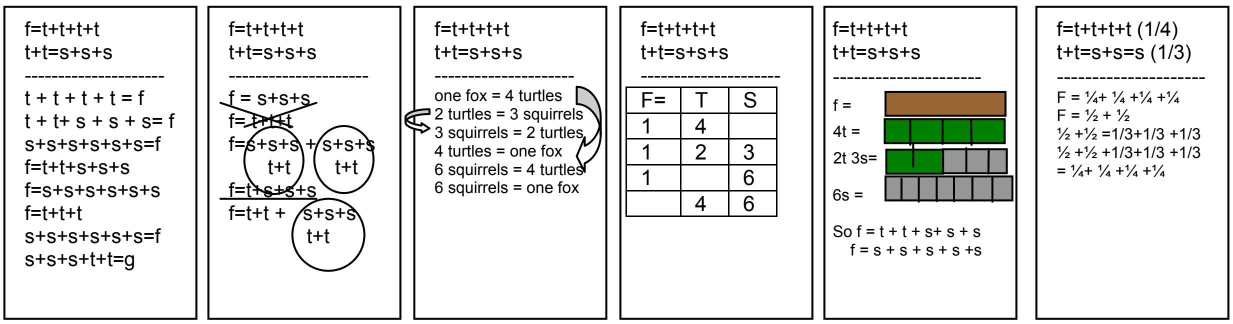
Reconvene the whole group and ask the groups to hang their charts on the board. Ask the groups to look at the charts and note how the solutions are the same or different. Possible anticipated student response may include:

"That group used the same symbols to represent the animals."

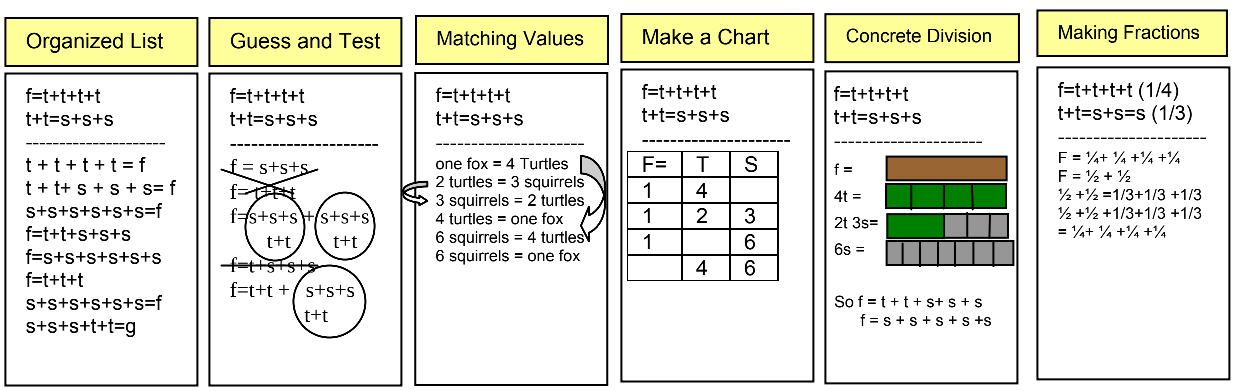
"These two groups both made the same number of equations."

"This group showed the same idea in a different way."

For example, if the group decided to organize by strategies the Bansho could be organized as below:



The teacher would listen to the groups describe their work and annotate on slips of paper the strategy the students used:



As the class discusses the Bansho, opportunities may arise to discuss other math concepts.

Rearranging the animals

Write the following equation on the board.

F + S = T + T + T + T + S

We know this equation is true because we are given that F = T + T + T + T. Adding a squirrel (S) to both sides maintains the balance. Now rearrange the letters on each side of the equation, as follows:

S + F = T + T + T + T + S

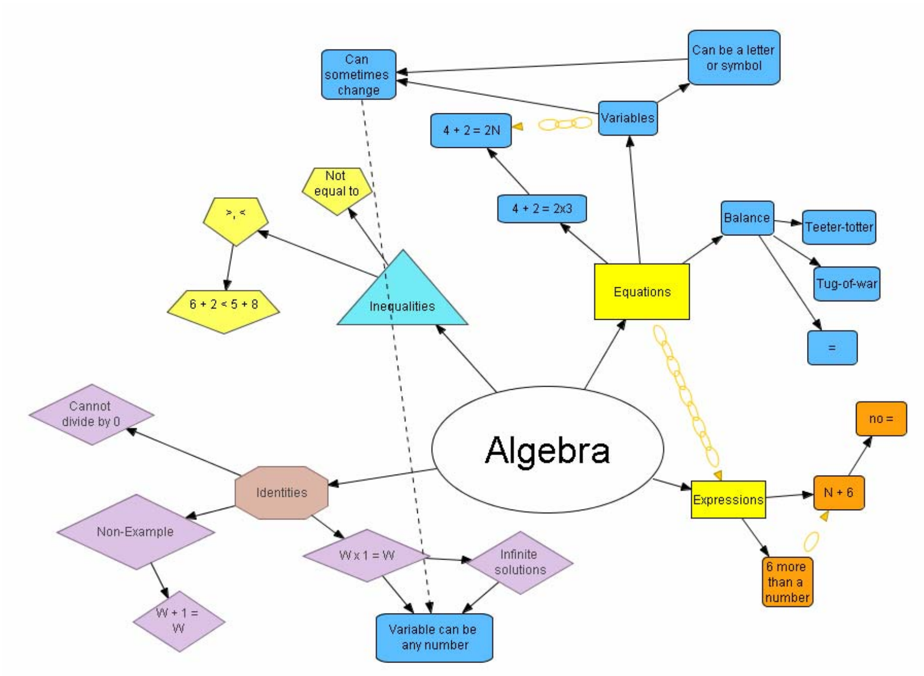
F + S = T + T + S + T + T

> Rearranging the order of the masses that are added together does not change their sum. For example, 3 + 5 = 5 + 3. This is the commutative property of addition. Does the commutative property also hold true for other operations? (It does for multiplication but not for subtraction or division.)

Ask students to consider whether the new equations are true. Have them work in pairs to rearrange the letters to create other equations. Share and discuss the new equations in a whole-class setting.

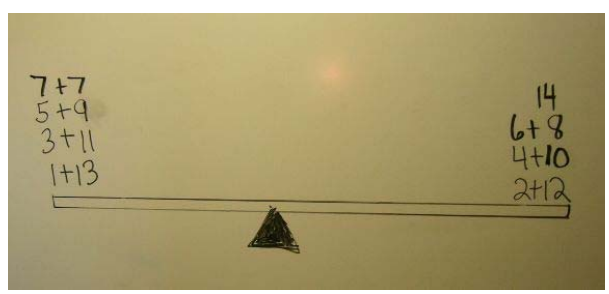
Reflecting and Connecting – (Part 3 of the 3 part lesson)

Create a word web wall with "Algebra" at the centre during the unit. Ask students to add connections after each lesson to the web. Below is a sample word web created using Smart Ideas™.



Balancing equations with numbers

Write the number 14 on the board. Then ask: "What numbers could I add to get 14?" Write students' suggestions on the board. Their suggestions will include 1 + 13, 2 + 12, 3 + 11, and so on. Draw a balance scale on the board and write 1 + 13 on one side. Then ask, "What could I put on the other side to create a balance?" Students may want to answer 14. While the answer is correct, ask for other possibilities (e.g., 3 + 11).



Discuss how 1 + 13 is equal to 3 + 11. Have students brainstorm further responses. Encourage them to see that they can use more than two addends (e.g., 1 + 3 + 10) and that they can use multiplication, division, or subtraction to create an infinite number of possibilities.

Finding missing numbers

Write the following equations on the board:

5 + U = 8 3 x 7 = T 36 ÷ R= 6 3 x N + 1 = 10

2 x T – 1 = 5

Ask students to work in pairs to find the unknown quantity in each equation.

Discuss the solutions and strategies with the class. Students may have used some of the following strategies:

Solutions:
5 + U = 8, **U = 3**
3 x 7 = T, **T = 21**
36 ÷ R= 6, **R = 6**
3 x N +1 = 10, **N = 3**
2 x T – 1= 5, **T = 3**

- *Guess and test*. Example: For 3 x N + 1 = 10, I guessed that N = 4 and tried it in the math sentence. It didn't work. So I tried a value of 3 for N and put it in the sentence. 3 x 3 + 1= 10 is correct.
- *Inverse operations*. Example: In 5 + U = 8, we can take 5 away from both sides to get U = 3. This type of reasoning (adding or subtracting something from both sides of an equation) was used in the tug-of-war equations.
- *Counting up.* Example: In 5 + U = 8, I counted 3 from 5 to 8.
- *Working backwards.* Example: In the question 2 x T – 1 = 5, I worked backwards. First I added 1 to 5 to get 6. Then I divided 6 by 2 and got 3. I then checked to see if I was right by substituting 3 for T and trying the math sentence. 2 x 3 – 1= 5 is correct.
- *Rephrasing the question.* Example: I said 36 "divided by what" = 6.

Tiered Instruction

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

Supports for student learning

- Students will benefit from being able to use concrete materials to model the relationships explored. For example, in the tug-of-war scenarios it would be helpful to use cut-outs of pictures of the animals, or else square tiles with letters (such as F for fox) to represent each of the animals.
- The summative activity described in the Assessment section is open-ended and can help meet the needs of all children.

Home Connection

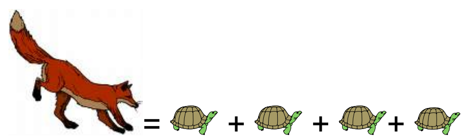
See **PA.BLM5a.3.**

Assessment

- Choose 1 or 2 equations prepared by students and ask students to explain their equation(s) in a mini interview.
- Ask students to give an example of an identity and to write an explanation of why it is an identity.
- Summative activity. Ask students to create their own tug-of-war scenario by picking the type of animals to include, deciding how many there should be of each animal, and making up relationships for their masses. Then ask students to pose 2 questions that may be answered based on their tug-of-war scenario.

PA.BLM5a.1 Fair Tug-of-War Teams

One fox, 6 squirrels, and 4 turtles go to the park to participate in a tug-of war game. The animals aren't sure who should participate in the tug-of war in order for the teams to be fair. A fox has the same mass of 4 turtles, and 2 turtles have the same mass at 3 squirrels. They use a teeter totter to check if each groups' mass are balancing equally.

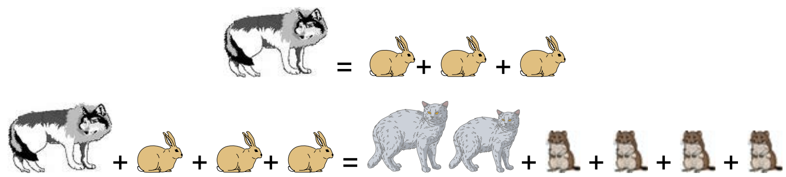


How many different ways can each side of the tug of war teams be organized?

PA.BLM5a.2 Teeter-Totter Equations

Extensions

A similar problem introduces a new set of animals, whose mass relationships are shown below:



Students are asked: “If the wolf, being a loner, wants to play on one side of the tug-of-war rope, how many bobcats and mice will be needed on the other side to make the game fair? Explain your reasoning.”

Teacher Notes:

For students to find a solution, they need to find a way of relating the two new equations:

$$W = R + R + R$$

$$W + R + R + R = B + B + M + M + M + M$$

They need to notice the following relationship:

$$W = R + R + R$$

$$W + R + R + R = B + B + M + M + M + M$$

Since 3 rabbits have the same mass as a wolf, we can regroup the right side to also be in two groups of equal masses:

$$W + R + R + R = B + M + M \quad + \quad B + M + M$$

This may result in students discovering that:

$$W = B + M + M$$

Many students will not use the formal reasoning described above. They will point and gesture, or might use diagrams or concrete materials, to communicate their ideas. Let the students express their thinking in their own words and in their own way but, where appropriate, model complementary strategies. For example, using equations made of letters that are coloured and circled or underlined to make relationships visually apparent is a strategy that all students should learn to use.

PA.BLM5a.3 Home Connection: A Massive Puzzle

Dear Parents/Guardians,

Your child has been working on the concept of "equality" in math sentences, and on the use of variables (letters and symbols) to represent a value. The class has explored the idea of a balance scale (teeter totter) to determine whether or not the two sides of a math sentence are equal (4 + 5 = 9). Using strategies such as logic, "guess and test", and "make a model", they solved a number of balance scale problems. Encourage your child to experiment with different strategies to solve the similar problem posed below and to check through substitution of values.

Find the mass of each object

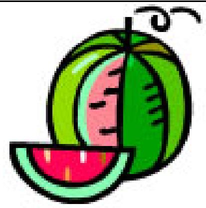
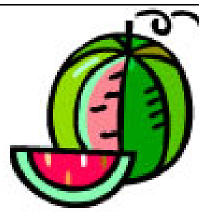
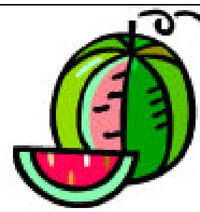
This problem deals with a bag of oranges (represented by O), four pineapples (P), and one watermelon (W). Use the clues in the table to find the mass of each object.

For example, using the first row we know that the combined mass of the oranges, pineapples, and watermelon is 17 kg. Use the other rows and columns to get more clues. Write an algebraic sentence to represent each relationship (e.g., O + P + W = 17). Explain how you solved the problem.

O

P

W

			17 kg
			14 kg
			21 kg
			13 kg
19 kg	22 kg	24 kg	

Your child can practise solving similar problems at www.mathplayground.com. Click on "Weigh the Wangdoodles".