Targeted Implementation & Planning Supports

GRADE 7 8 9 Applied MATHEMATICS

TIPS for Grades 7, 8, and 9 Applied Math is designed to be useful to teachers in both Public and Catholic schools, and is intended to support beginning teachers, provide new insights for experienced teachers, and help principals and professional development providers as they work to improve mathematics education.

Any references in this document to particular commercial resources, learning materials, equipment, or technology reflect only the opinions of the writers of this resource, and do not reflect any official endorsement by the Ministry of Education or by the Partnership of School Boards that supported the production of the document.

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Targeted
Implementation &
Planning
Supports

Mathematics

Grades 7, 8, and 9 Applied

1

Developing Mathematical Literacy

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Developing Mathematical Literacy

A. Introduction

Targeted Implementation and Planning Supports (TIPS) offers ways of thinking about mathematics education, resources, and teacher education for those working with students in Grades 7 to 9. "**In this changing world, those who understand and can do mathematics have significant opportunities and options for shaping their future."** NCTM 2000 p. 5.
Students in this age group are at a critical, transitional stage where their perceptions of mathematics will help to shape their success in secondary mathematics and their career decisions.

"In this changing world, those who understand and can do mathematics have significant opportunities and options for shaping their future."
NCTM 2000 p. 5

Mathematics competence and confidence open doors to productive futures. Implementation of this resource will help to enhance the mathematics knowledge and understanding of students in Grades 7 to 9, and help them develop the critical skills identified in the Conference Board of Canada's Employability Skills Profile: communicate, think, continue to learn throughout their lives, demonstrate positive attitudes and behaviours, responsibility, and adaptivity, and work with others.
One way to think of a person's understanding of mathematics is that it exists along a continuum. At one end is a rich set of connections. At the other end of the continuum, ideas are isolated or viewed as disconnected bits of information. (Skemp) **A sound understanding of mathematics is one that sees the connections within mathematics and between mathematics and the world.** However, to lead students to rich connections, "teachers must be agents of change that they did not experience as students" (Anderson, D. S. & Piazza, J. A., 1996.)

A sound understanding of mathematics is one that sees the connections within mathematics and between mathematics and the world.

The intent is to help both students and teachers develop the "big picture" of mathematics that includes: competence in mathematical skills, substantive understanding of mathematical concepts, and the application of these skills and understandings in problem-solving situations. This resource package helps teachers to see how, in their program and daily lesson planning, they can address the critical **connections** between:

- instruction and assessment;
- one mathematics topic and the next;
- one strand and the other strands;
- the mathematics done in class and students' sense-making processes;
- learning mathematics and doing mathematics;
- the instructional strategy selected for a specific learning goal and research into how students learn mathematics;
- students' prior learning and new knowledge and understanding;
- existing resources and the envisioned program;
- the mathematics classroom and home;
- mathematics topics and topics in other disciplines.

This resource is intended to:

- support beginning teachers;
- provide new insights for experienced teachers;
- help principals and professional development providers as they work to improve mathematics education for young people.

This package is a ‘work in progress.’ It will be revised, refined and added to, as teams of teachers come together to engage in collaborative planning, then share their work. It is available electronically at www.curriculum.org

Key Messages

There are important messages in TIPS for the many stakeholders involved in the mathematical education of the young adolescent:

TIPS for Teachers

- Focus on important mathematical concepts or “big ideas” that cluster expectations.
- Include a variety of instructional strategies and assessment strategies.
- Value the abilities and needs of the adolescent learner.
- Provide a positive environment for learning mathematics through problem solving.
- Allow opportunities for students to explore, investigate, and communicate mathematically as well as opportunities to practise skills.
- Encourage a variety of solutions that incorporate different representations, models, and tools.
- Incorporate relevant Ontario Catholic School Graduate Expectations.

TIPS for Principals

- An effective mathematics program should have a variety of instructional and assessment strategies.
- Effective mathematics classrooms are active places where you can hear, see, and touch mathematics.
- Principals are encouraged to provide teachers with necessary support for an effective mathematics program. This can be done through:
  - student and teacher resources;
  - providing time for teachers to collaborate and share ideas;
  - providing in-service opportunities and information about mathematics education for teachers;
  - celebrating mathematics learning for all students, not just the achievements of a select few;
  - showing a positive attitude towards mathematics;
  - demonstrating the importance of life-long learning in mathematics.

TIPS for Coordinators

- Mathematics teachers need support.
- All teachers require information and in-service about mathematics education.
- Teachers need to know how and why these activities promote student learning of mathematics.

TIPS for Researchers

Use of this package may bring to light a number of research questions focused on the topics that were the subjects of literature reviews.
Synopses of research on a wide variety of themes are included in the Research section (p. 17). Any of these could be expanded into a more complete literature review with accompanying research questions and follow-up research. It is hoped that Ontario-based research that addresses any of the elements featured in *Targeted Implementation and Planning Supports (TIPS)* will be submitted to the project for inclusion and reference. Please contact the Ontario Curriculum Centre at occ@curriculum.org, if you have research that you would like to contribute.

Program Planning Supports

This package provides grade-specific and cross-grade program planning supports. These are structured to provide opportunities for in-depth, sustained interaction with key mathematical concepts, or "big ideas" that cluster and focus curriculum expectations. The intent is to help students see mathematics as an integrated whole, leading them to make connections and develop conceptual and relational understanding of important mathematics.

Content and Reporting Targets by Grade

The package for each grade begins with a one-page overview of the mathematics program that includes a suggested sequencing of curriculum expectation clusters, rationale for the suggested sequence, and in Grades 7 and 8, suggested strands for reporting in each term. Detailed alignment of all curriculum expectations with the identified clusters is included in the Appendix for each grade level.

Continuum and Connections across Grades

Continuum and Connections, Section 2 is based on significant themes in mathematics and includes *Mathematical Processes* and five content-based packages – *Patterning to Algebraic Modelling, Solving Equations and Using Variables as Placeholders, Developing Perimeter and Area Formulas, Integers*, and *Fractions.*

In *Mathematical Processes* – Section 2

- The first page situates the mathematical processes in a larger context and asks key questions about development of the processes – through the grades, for different types of learners, and using different strategies.
- To answer these key questions, there is a description of student and teacher roles and assessment suggestions for each of the four mathematical processes. These processes and the research that lead to their identification are outlined in *Theoretical Framework for Program Planning*, p.10 and *Research*, p. 17.

For each content-based package – Section 2

- The first page situates the mathematical theme in a larger context, shows connections to other subjects, careers, and authentic tasks, and identifies manipulatives, technology, and web-based resources useful in addressing the theme.
- *Connections Across Grades* outlines scope and sequence, using before Grade 7, Grade 7, Grade 8, Grade 9, and Grade 10 as organizers.
- *Instruction Connections* suggests instructional strategies, with examples, for each of Grade 7, Grade 8, and Grade 9 Applied, and includes advice on how to help students develop understanding.
- *Connections Across Strands* provides a sampling of connections that can be made across the strands, using the theme as an organizer.
- *Developing Proficiency* presents sample tests and *Developing Mathematical Processes*, a set of four short-answer questions – one per mathematical process – based on the theme for each of Grade 7, Grade 8, and Grade 9 Applied are included.
- Questions posed in *Extend Your Thinking* and *Is This Always True*? help students develop depth of understanding. Answers show a variety of representations and strategies that students may use and that teachers should validate.

Summative Tasks

Summative tasks are provided for each of Grade 7, Grade 8, and Grade 9 Applied mathematics. These tasks focus on some of the major concepts of the mathematics program and require facility in several strands that the students have studied throughout the program.

Teachers can use these summative tasks to design other tasks in the program and to recognize some of the important ideas in the program. Teachers can use the summative tasks in planning a program by determining what important mathematical ideas students should understand by the end of the program and then designing the instructional approaches that support those mathematical ideas.

Lesson Planning Supports

- The pace and structure of lessons will vary depending on the goals of the lesson. Using a single sequence of elements, such as taking up homework, teacher demonstration, student practice can be uninspiring or monotonous.
- Mathematics classes should:
  - provide opportunities for students to become proficient in basic skills;
  - focus on introducing new skills and concepts through problem solving;
  - incorporate a balance and variety of teaching strategies, assessment strategies, student groupings, and types of activities.
- Each lesson should appeal to auditory, kinesthetic, and visual learners. Appealing to all these preferences deepens the understanding of all students.
- Lessons should introduce algorithms only after students have constructed meaning for the procedure, and introduce formulas only after students develop a conceptual understanding of the relationships among measurements.
- Lessons are developed in a template to outline a desired vision for classes.

Lesson Planning Template

The following organizers in the template provide for concise communication while allowing the teacher to vary the pace and types of groupings and activities for classes:

Minds on… suggests how to get students mentally engaged in the first minutes of the class and establishes a positive classroom climate, making every minute of the math class count for every student.

Action! suggests how to group students and what instructional strategy to use.

This section maps out what students will do and how the teacher can facilitate and pose thought-provoking questions. Suggestions for time management, scaffolding, and extension are included, as appropriate.

Suggested groupings and strategies include:

<u>Groupings:</u>

Carousels, Expert Groups, Groups of 4, Individual, Jigsaw, Pairs, Pair/Share, Small Groups, Think/Pair/Share, Whole Class.

<u>Strategies:</u>

Acting, Brainstorm, Concept Map, Conferencing, Connecting, Demonstration, Discussion, Experiment, Field Trip, Game, Guest Speaker, Guided Exploration, Independent Study, Interview, Investigation, Kinesthetic Activity, Model Making, Note Making, Portfolio, Practice, Presentation, Problem Solving, Reflection, Research, Response Journal, Retelling, Role Playing, Simulation, Survey, Tactile Activity, Visual Activity, Worksheet.

The time clock/circle graphic (Grades 7 and 8) and the time bar graph (Grade 9) suggest the proportion of class time to spend on various parts of the mathematics class.

Consolidate/Debrief suggests ways to 'pull out the math,' check for conceptual understanding, and prepare students for the follow-up activity or tomorrow's lesson. Often this involves whole class discussion and sharing. Students listen to and contribute to reflections on alternate approaches, different solutions, extensions, and connections. Students should be well prepared to do mathematics individually after the three-part lesson.

H

Home Activity or Further Classroom Consolidation suggests meaningful and appropriate follow-up. These activities provide opportunities:
- to consolidate understanding;
- to build confidence in doing mathematics independently;
- for parents to see the types of math activities students engage in during class and to see connections between the mathematics being taught and life beyond the classroom;
- for giving students some choice through differentiated activities.

- The **MATCH** (**M**inds on, **A**ction, **T**iming, **C**onsolidate, **H**ome Activity or Further Classroom Consolidation) acronym reminds teachers that all of these elements must be considered in lesson planning. It suggests that connection/matches should be made in each lesson to:
  - program goals;
  - research and effective instruction;
  - characteristics of adolescent learners.

Interpreting the Lesson Outline Template

Download the Lesson Outline Template at www.curriculum.org/occ/tips/downloads.shtml

Lesson Outline: Days 5 – 9

Sequence of Lessons Addressing a Theme

Grade 7

Grade Level

BIG PICTURE

Students will:
- explore and generalize patterns;
- develop an understanding of variables;
- investigate and compare different representations of patterns.

Lessons are planned to help students develop and demonstrate the skills and knowledge detailed in the curriculum expectations.
- **To help students value and remember the mathematics they learn, each lesson is connected to and focussed on important mathematics as described in the Big Picture.**
- **Since students need to be active to develop understanding of these larger ideas, each point begins with a verb.**
- **Sample starter verbs: represent, relate, investigate, generate, explore, develop, design, create, connect, apply**

Day	Lesson Title	Description	Expectations
5	Toothpick Patterns	• Review patterning concepts • Build a growing pattern • Explore multiple representations	7m70, 7m72 CGE 3c, 4f
6	Patterns with Tiles	• Build a pattern • Introduce the *n*th term	7m66, 7m71 CGE 4b
7	Pattern Practice	• Continued development of patterning skills	7m67, 7m71, 7m75 CGE 2c, 5e
8	Pattern Exchange	• Class sharing of work from previous day.	7m69, 7m75 CGE 2c, 5e
9	Performance Task	• Performance Task - individual	7m66, 7m67, 7m73, 7m75 CGE 5g

A brief descriptive lesson title

List curriculum expectations (and CGEs)

- **Two or three points to describe the content of this lesson.**
- **Points begin with a verb.**
- **Individual lesson plans elaborate on how objectives are met.**

NOTES

a) While planning lessons, teachers must judge whether or not pre-made activities support development of big ideas and provide opportunities for students to understand and communicate connections to the "Big Picture."

b) Ontario Catholic School Graduation Expectations (CGEs) are included for use by teachers in Catholic Schools.

c) Consider auditory, kinesthetic, and visual learners in the planning process and create lessons that allow students to learn in different ways.

d) The number of lessons in a group will vary.

e) Schools vary in the amount of time allocated to the mathematics program. The time clock/circle on completed Grade 7 and 8 lessons suggests the fractions of the class to spend on the Minds On, Action!, and Consolidate/Debrief portions of the class. Grade 9 Applied lessons are based on 75-minute classes.

f) Although some assessment is suggested during each lesson, the assessment is often meant to inform adjustments the teacher will make to later parts of the lesson or to future lessons. A variety of more formal assessments of student achievement could be added.

Interpreting the Lesson Planning Template

Download the Lesson Planning Template at www.curriculum.org/occ/tips/downloads.shtml

Day #: Lesson Title

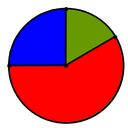
Grade level

Day 1: Encouraging Others

Materials used in the lesson

Grade 8

Same two or three objectives listed in the lesson outline



Time colour-coded to the three parts of the day's lesson

Description

- Practise the social skill of encouraging others.
- Identify strategies involving estimation problems.
- Set the stage for using estimation as a problem-solving strategy.

Materials

- BLM 1.1
- birdseed

Suggested student grouping → teaching/learning strategy for the activity.

Assessment Opportunities

Minds On ...

Whole Group → Brainstorm

Explain why it is important to encourage others. Explicitly teach the social skill, "Encouraging Others," through a group brainstorm. Create an anchor chart using the criteria: What does it look like? What does it sound like?

"Learning is socially constructed; we seldom learn isolated from others."
– Bennett & Rolheiser

- Mentally engages students at start of class
- Makes connections between different math strands, previous lessons or groups of lessons, students' interests, jobs, etc.
- Introduces a problem or a motivating activity
- Orients students to an activity or materials.

Action!

Think/Pair/Share → Gather Data

Use an overhead of the Think/Pair/Share process (TIP 2.1) and student copies of BLM 1.1. Students gather data.

Learning Skill/Observation/Mental Note: Circulate, observing social skills and listening to students.

Indicates an assessment opportunity
- what is assessed/strategy/scoring tool

Consider using stickers as a recognition for examples of the social skill being applied by a group.

- Students do mathematics: reflecting, discussing, observing, investigating, exploring, creating, listening, reasoning, making connections, demonstrating understanding, discovering, hypothesizing
- Teachers listen, observe, respond and prompt

Share with students some of the positive words and actions observed during the activity and invite students to make additions to the anchor chart on Encouraging Others.

Indicates suggested assessment

Whole Class → Sharing

Based on 'teachable topics' during the Think/Pair/Share Activity, e.g., a particularly effective phrase/statement expressed by a student, clarification of the cooperative learning strategy, an interesting result on BLM 1.1, ask representatives of groups to share their results or report on their process.

Solving Fermi problems is a way to collect diagnostic assessment data about social skills, academic understandings and attitudes towards mathematics (see TIP 1.2).

Consolidate Debrief

"Pulls out´ the math of the activities and investigations

Whole Class → Discussion

Use the posters Inquiry Model Flow Chart, Problem-Solving Strategies, and Understanding the Problem. Discuss how these posters will be of assistance over the next few days as well as during the whole math program. Point out that when students encourage others, it makes it safe for them to try new things and contribute to group activities.

Tips for the Teacher
These include:
- instructional hints
- explanations
- background
- references to resources
- sample responses to questions/tasks

Prepares students for Home/Further Classroom Consolidation

Social Skill Practice Reflection

Home Activity or Further Classroom Consolidation

Interview one or more adults about estimation using the following guiding questions and record your responses in a math journal. Summarize what you notice about the responses. You may be asked to share this math journal entry with the class.

Focus for the follow-up activity.

Meaningful and appropriate follow-up to the lesson.

Research Base for Planning Templates

Each of the organizers in *Continuum and Connections* (Section 2) and *Lesson-Planning Templates*, p.6-7 is supported by a one-page research synopsis. It is through a thoughtful combination of decisions that teachers proactively establish *Classroom Management* (p. 27) and a community of learners and demonstrate their commitment to equity. **It is important to set high standards for all students and to provide the supports and extensions that help all students learn.**

It is important to set high standards for all students and to provide the supports and extensions that help all students learn.

By making thoughtful choices at each decision point in the templates, teachers demonstrate their understanding of *Adolescent Mathematics Learners* (p.26) as it relates to the students in their class.

Time

Although the time allotment for mathematics classes may vary within and between the elementary and secondary panels and between schools, teachers must plan appropriate time for the various components of each lesson. To assist teachers in this stage of planning, research regarding *Japanese Lesson Structure* (p. 33) is provided as an example of an internationally respected model of student inquiry and related time allotment.

Materials

The contemporary mathematics classroom features a combination of traditional materials and more recently developed tools. *Concrete Materials* (p. 29) provides a description and analysis of effective mathematical manipulatives. *Technology* (p. 41), teachers can read about the various types and applications of technological tools, and important considerations relating to their implementation.

Type of Learner

Research regarding the complex amalgam of students' abilities and competencies, e.g., Gardner's Multiple Intelligence research, has had an immense impact on modern educational thinking. Students not only demonstrate many different kinds of skills and creative talents, but possess different learning styles including auditory, visual, and kinaesthetic. The lesson template encourages teachers to mindfully interpret each lesson in terms of these considerations.

Grouping

Research has shown that students benefit from a balance of different learning structures. *Flexible Grouping* (p. 31) informs teachers of the many options available for logistical planning within the classroom. These options include heterogeneous pairs and small groups, homogeneous pairs and small groups, co-operative learning activities, independent research or class work, together with whole-class and small group discussions.

Instructional Strategies

A variety of effective instructional strategies are explored within this document. *Differentiated Instruction* (p.30) describes how teachers can orchestrate different activities for different students simultaneously, for one or more instructional purposes, and to the benefit of all involved. *Mental Mathematics and Alternative Algorithms* (p. 34) provides insight into the research surrounding estimation and algorithmic exploration. Further valuable strategies such as *Scaffolding* (p. 38), *Student-Centred Investigations* (p. 39), and *Teacher-Directed Instruction* (p. 40) are also detailed.

Communication

Communication contributes to development of mathematical understanding in the classroom. *Metacognition* (p. 35) describes ways in which teachers can encourage students to analyze and alter their own thinking and learning processes. *Questioning* (p. 37) discusses the complexity involved in, skills required for, and examples of higher-order questioning. In *Communication* (p. 28), a model is presented for a positive classroom environment in which students are encouraged to question, comment, theorize, discuss, and defend ideas.

Assessment

The purpose of assessment is to improve student learning by *Providing Feedback* (p. 36) and using assessment data to inform and guide instruction. *Principles of Quality Assessment* (p. 21) and *Linking of Assessment and Instruction* (p. 22), describe assessment that models and is embedded in instructional planning connects to student needs and prior knowledge, and promotes achievement. Good assessment practices provide opportunities for students to demonstrate what they "know and can do" and promote their success. Using a variety of assessment strategies and tools assists in providing *Balanced Assessment* (p. 23) that recognizes that students demonstrate their understanding in many different ways and that provides a complete picture of a student's mathematical understanding. Students who have difficulty in mathematics may need adjustments or *Assessment Accommodations* (p. 24) opportunities to demonstrate achievement. *Evaluation* (p. 25) involves the judging and interpreting of assessment data and the assigning of a grade. Evaluation should be a measure of a student's current achievement in the important mathematical concepts in the program.

Developing Understanding

The facilitation of conceptual understanding as it relates to geometry, is elaborated in a discussion of the *van Hiele Model* (p. 42). The importance of conceptual understanding is also highlighted in each section of *Continuum and Connections*.

Home Activity or Further Classroom Consolidation

Within the *Providing Feedback* (p. 36) section, teachers are reminded of the importance of consistent and detailed feedback regarding student progress to both students and parents throughout the school year. The lesson template encourages teachers to plan a variety of activities for follow-up just as they plan a variety of lesson types, depending on the learning goal. **Students need opportunities to learn, practise, select their own preferred problem-solving methods and strategies, and demonstrate learning in a variety of ways.**

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B. Theoretical Framework for Program Planning

Vision

What is mathematical literacy and why it is important?

Mathematical literacy, as defined by the Programme for International Student Assessment (PISA, www.pisa.oecd.org/knowledge/summary/b.html) is “measured in terms of students’ capacity to:

- recognise and interpret mathematical problems encountered in everyday life;
- translate these problems into a mathematical context;
- use mathematical knowledge and procedures to solve problems;
- interpret the results in terms of the original problem;
- reflect on the methods applied; and
- formulate and communicate the outcomes.”

Another source suggests that:

> “Mathematics literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen.” (*Measuring Up*, OECD PISA Study, 2001, p. 10)

Mathematical literacy is valued for many different reasons. Mathematics provides powerful numeric, spatial, temporal, symbolic, and communicative tools. Mathematics is needed for “everyday life” to assist with decision making. As John Allen Paulos pointed out in his book, *Innumeracy* (2001), “ … numeracy is the ability to deal with fundamental notions of number and chance in order to make sense of mathematical information presented in everyday contexts.” Mathematics and problem solving are needed in the workplace for many professions such as health science workers or graphic artists, as well as statisticians and engineers. Mathematics is also, ultimately, a cultural and intellectual achievement of humankind and should be understood in its aesthetic sense (NCTM, 2000). All people have a right of access to the domain of mathematics.

The need for mathematical literacy for all students requires us to describe what an effective mathematics program looks like – one that fosters mathematical literacy in all students.

What does effective mathematics teaching and learning look like?

There are several components to sound mathematics teaching and learning:

a) a classroom climate that is conducive to learning
b) mathematical activities that engage students in important mathematical concepts
c) a focus on problem solving
d) a program that provides a balance of instructional and assessment strategies that sustains mathematical understanding.

a) A Classroom Climate for Mathematics Learning

Learning does not occur by passive absorption of information. Students approach a new task with prior knowledge, assimilate new information, and construct new understandings through the task (Romberg, 1995). The classroom climate must be conducive to such learning. The following factors need to be considered:

- Prior Knowledge: Students come to class with significant but varying prior knowledge. Such prior knowledge should be valued and used during instruction.
- Equity: All students are able to learn mathematics. Excellence in mathematics requires high expectations and strong support for all students.
- Technology: Technology influences the mathematics that we teach and how we teach it. To this end, teachers need to keep up-to-date with technology.

- Social, emotional, and physical considerations: Brain research on learning (Given, 2002) suggests that there are five "theatres of the mind" — emotional, social, physical, cognitive, and reflective. The emotional, social, and physical learning systems tend to be the most powerful in terms of their demands. The level of their functioning determines how effectively the cognitive and reflective systems operate.
- Attitude and confidence: Fostering a positive attitude and building student confidence helps students to develop values that do not limit what they can do or can assimilate from a learning experience. Students' emotions, beliefs, and attitudes towards mathematics and learning interact with cognition and are instrumental in empowering learners to take control of their own learning and express confidence in their mathematical decisions.

A classroom that is conducive to learning takes all of these areas into consideration so that students have a safe and positive environment for learning. Traditionally, mathematics programs have attended to the cognitive learning system with less attention being paid to the other four learning systems described above. However, mathematics teachers are becoming more aware of the importance of the affective domain – emotional, social, and reflective learning systems. For instance, curriculum policy requires that students learn how to communicate effectively in mathematical contexts. This communication provides perspectives into students' feelings and values as well as their thinking. **The intent of *Targeted Implementation and Planning Supports (TIPS)* is to help teachers activate all the student's learning systems through the delivery of a balanced mathematics program.**

An appropriate classroom environment in Grade 7, Grade 8, and Grade 9 Applied also considers the characteristics of the adolescent learner. These characteristics include intellectual development such as the ability to form abstract thought and make judgments. Adolescents are going through many physical changes, and may be experiencing emotional uncertainty. Recognition by peers and social status are extremely important to them. In working with adolescents, it is important to recognize signs of disengaged or at-risk learners and to provide support for them. A more extensive discussion of the adolescent learner is included in *Adolescent Mathematics Learners*, p. 26.

The intent of *Targeted Implementation and Planning Supports (TIPS)* is to help teachers activate all of the student's learning systems through the delivery of a balanced mathematics program.

Summary

A classroom environment that supports student learning has the following qualities:

- Students are supported to take risks, explore different problem-solving strategies, and communicate their understanding.
- Mathematics is seen, heard, and felt.
- The teacher models and promotes a spirit of inquiry.
- Students' prior knowledge is valued and built upon.
- Students actively explore, test ideas, make conjectures, and offer explanations.
- Social skills are developed to promote effective teamwork.

b) Mathematical Activity

Mathematical activity should develop mathematical thinking. The successful teaching of mathematics can be characterized as helping students develop mathematics proficiency. *In Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001), the concept of mathematical proficiency is described in terms of five intertwining components:

- *conceptual understanding* – comprehension of mathematical concepts, operations and relations
- *procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence* – ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
- *productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

These five competencies capture what it means for someone to learn mathematics successfully. The integrated and balanced development of all five should guide the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn, on one hand, solely by internalizing what a teacher or book says, or, on the other hand, solely by inventing mathematics on their own. (Kilpatrick et al., 2001, p. 11)

"…conceptual understanding without skills is inefficient."
(*Mathematics Program Advisory*, June 1996)

Mathematical activity should be balanced with time to investigate and explore as well as time to practise skills. As such, mathematical activity can be organized around five different processes as outlined below:

- Knowing Facts and Procedures: **"…conceptual understanding without skills is inefficient."** (*Mathematics Program Advisory*, June 1996) There are basic facts and proficiencies required at each grade level to develop mental mathematics skills including recognizing the reasonableness of a result and the use of symbolic manipulation software and calculators. Some procedures involve manipulation or the performance of calculations and the successful use of algorithms and procedures.
- Reasoning and Proving: Reasoning is essential to mathematics. Developing ideas, making conjectures, exploring phenomena, justifying results, and using mathematical conjectures help students see that mathematics makes sense. As teachers help students learn the norms for mathematical justification and proof, the repertoire of types of reasoning available to students – algebraic and geometric reasoning, proportional reasoning, probabilistic reasoning, statistical reasoning, and so forth – expand (NCTM, 2000). Using manipulatives and technology efficiently and effectively in investigating mathematical ideas and in finding solutions to mathematical problems provides students with problem-solving tools and a way to create a visual representation.
- Communicating: Communication plays an important role in supporting learners by clarifying, refining, and consolidating their thinking. Mathematically literate learners should be able to communicate their mathematical ideas orally and in writing while defending and offering justification for such ideas.
- Making Connections: **"Skills without conceptual understanding are meaningless."**(Mathematics Program Advisory, June 1996) Students demonstrate their understanding through application of knowledge and skills and through problem solving. Component skills of application and problem solving are representing, selecting, and sequencing procedures. Representing involves the learner in constructing and alternating between various mathematical models such as equations, matrices, graphs, and other symbolic and graphical forms.

"Skills without conceptual understanding are meaningless."
(*Mathematics Program Advisory*, June 1996)

- Valuing Mathematics: Values include learners' emotions, beliefs, and attitudes towards mathematics and learning. Such affective processes interact with cognition and are instrumental in empowering the learner to express confidence in their mathematical decisions and to take control of their own learning.

It is important to provide students with opportunities to practise these activities individually, and to provide rich and challenging problems that require students to combine these activities in various ways.

Summary
Mathematical activity in a mathematics classroom is characterized by:

- Computing
- Recalling facts
- Manipulating
- Using manipulatives and technology
- Exploring
- Hypothesizing
- Inferring/concluding
- Revising/revisiting/reviewing/reflecting
- Making convincing arguments, explanations, and justifications
- Using mathematical language, symbols, forms, and conventions
- Explaining
- Integrating narrative and mathematical forms
- Interpreting mathematical instructions, charts, drawings, graphs
- Representing a situation mathematically
- Selecting and sequencing procedures

c) A Focus on Problem Solving

Problem solving is fundamental to mathematics. "**Without problem solving, skills and conceptual understanding have no utility.**"(*Mathematics Program Advisory*, June 1996) The mathematical processes can be aligned with various problem-solving models, with the mathematical competencies as defined by Kilpatrick, Swafford, and Findell (2001) and with the categories of the Achievement Chart in the Ontario Ministry of Education Mathematics Curriculum. This illustrates that the many different types of organizing structures can be aligned so that teachers see how they fit together and address similar processes of mathematical thinking.

"Without problem solving, skills and conceptual understanding have no utility."
(*Mathematics Program Advisory*, June 1996).

The categories in the elementary Achievement Chart – Problem Solving, Understanding of Concepts, and Application of Procedures – and the categories in the secondary Achievement Chart – Knowledge/Understanding, Application, and Thinking, Inquiry, and Problem Solving – should not be viewed as isolated but rather as component processes of problem solving. Problem solving involves designing a plan and applying a variety of skills to solve the problem. The problem-solving process also requires mathematical communication. Thus, problem solving is seen as the most comprehensive framework in the learning of mathematics. Mathematical activity and learning should be centred on problem solving. NCTM's *Principles and Standards* document (2000) points out the importance of problem solving:

"Solving problems is not only a goal of learning mathematics but also a major means of doing so."
NCTM's *Principles and Standards* (2000)

> **"Solving problems is not only a goal of learning mathematics but also a major means of doing so.** Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas ... Good problems will integrate multiple topics and will involve significant mathematics." (p. 52).

Students need regular opportunities to represent, manipulate, and reason in the context of solving problems and conducting mathematical inquiries. It is expected that students use a variety of tools, including technology to solve problems and develop their understanding. Students should regularly communicate their thinking to peers and to the teacher and reflect on their learning. Problem solving is the most effective way to engage students in all of the mathematical processes. In *Elementary and Middle School Mathematics*, John A. Van de Walle says:

> "Most, if not all, important mathematics concepts and procedures can best be taught through problem solving. That is, tasks or problems can and should be posed that engage students in thinking about and developing the important mathematics they need to learn." (p. 40)

d) A Balanced Mathematics Program

In a balanced mathematics program, students become proficient with basic skills, develop conceptual understanding, and become adept at problem solving. All three areas are important and need to be included. Students need to develop both skills and conceptual understanding: problem solving is the medium for students to use to make connections between skills and conceptual understanding (*Mathematics Program Advisory*, 1996).

This resource gives teachers examples of lessons and assessments that will allow them and their students to experience a balanced mathematics program. Once experienced, a balanced mathematics program empowers both teachers and students as life-long learners of mathematics.

Summary

A balanced mathematics program has:

- students working in groups, pairs, and individually
- a variety of activities for students to engage in all of the mathematical processes
- a variety of diagnostic, formative, and summative assessment data to improve student learning and adjust program
- a focus on developing key mathematical concepts or "Big Ideas"

Making It Happen

This section highlights some of the student and teacher roles in a mathematics class, and describes the components of an effective lesson.

As problem solving places the focus on the student's attention to ideas and making sense, student roles are very active. Students are encouraged to:

- make conjectures;
- gather data;
- explore different strategies;
- share their ideas;
- challenge and defend ideas and solutions;
- consolidate and summarize their understanding;
- practise skills that relate to the mathematical concepts being explored.

Teachers' roles are much more than telling and explaining. Teachers need to:

- create a supportive mathematical environment;
- pose worthwhile mathematical tasks;
- recognize when students need more practice with skills and accommodate those needs;
- encourage mathematical discussion and writing;
- promote the justification of student answers;

- pose important questions that help students “pull out the math;”
- listen actively to student questions and responses and react accordingly;
- consolidate and summarize important mathematical concepts and help students make connections.

The structure of a lesson in mathematics can take a variety of forms. In a lesson that focuses on problem solving, the following components incorporate the mathematical processes that have been discussed:

- Teacher poses a complex problem.
- Students work on the problem.
- Students share solutions.
- Teacher consolidates understanding and summarizes discussion.
- Students work on follow-up activities.

In a lesson focusing on skill development, the following sequence should motivate the hard work that is sometimes needed to develop proficiency:

- Teacher poses an interesting problem that will extend a student’s current skill development.
- The problem exposes the need for a particular skill.
- Building on students’ prior knowledge, the teacher instructs, guides, or demonstrates the new skill.
- The teacher guides students through the solution of the original problem by applying the new skill.
- Students practise the skill and apply the skill to similar problems.

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C. Research – Topic Summaries

- Assessment and instruction should reflect the four categories of the Achievement Chart or actions of mathematics, as well as mathematical competences.
- One way to plan purposeful assessment is to include it when planning a lesson, unit, and course.

- An assessment plan that is well-aligned with the mathematics curriculum and instruction is built around tasks that are similar to the tasks that form instruction.
- As teachers deepen their understanding of assessment, they will find that almost any setting can be an opportunity for assessment.

- Teachers should provide assessment tasks that include mathematical content and mathematical processes such as reasoning, problem solving, and communication.
- Assessment tasks should vary in terms of task type, openness, length, modes of presentation, modes of working (groupings), and modes of response.

- Teachers may need to adapt their assessment plan to better suit the characteristics and circumstances of students in the class.
- A variety of adaptations, including accommodations, can be used to support at-risk students.

- Evaluation should represent a current and accurate picture of a student's achievement of the curriculum expectations.
- Evaluation should represent a balance of strands and Achievement Chart categories, and focus on the important mathematical concepts in the course.

- Adolescent students learn mathematics best in environments which allow for physical activity, social interaction, technological investigations, choice, variety, and meaningful input.
- Patience and awareness are key factors in providing adolescent learners with the kind of emotional and pedagogical support that they require during this time of personal change.

- Good classroom management results from well planned lessons and well established routines.
- A stimulating classroom that engages students through a balanced variety of teaching, learning, and assessment strategies prevents and reduces behaviour problems.

- Communication in mathematics requires the use and interpretation of numbers, symbols, pictures, graphs, and dense text.
- When explaining their solutions and justifying their reasoning, students clarify their own thinking and provide teachers with windows into students' depth of understanding.

- Effective use of manipulatives helps students move from concrete and visual representations to more abstract cognitive levels.
- Teachers can diagnose gaps in a student's conceptual understanding through observing the use of manipulatives and listening to the accompanying narrative explanation.

Mathematics Reform

Mathematics education is undergoing significant reform in many countries, including Canada and the United States. *Principles and Standards for School Mathematics* (NCTM, 2000) highlighted six key principles—Equity, Curriculum, Teaching, Learning, Assessment, and Technology—that must guide meaningful curricular reform efforts. The vision statement which prefaced the document elaborated on the need for the continued improvement of mathematics education:

> Evidence from a variety of sources makes it clear that many students are not learning the mathematics they need or are expected to learn (Kenney and Silver, 1997; Mullis et al., 1997, 1998; Beaton et al., 1996). The reasons for this deficiency are many. In some instances, students have not had the opportunity to learn important mathematics. In other instances, the curriculum offered to students does not engage them. Sometimes students lack a commitment to learning. The quality of mathematics teaching is highly variable. There is no question that the effectiveness of mathematics education in the United States and Canada can be improved substantially.
> (2000, p. 5)

Similarly, *The Ontario Curriculum: Mathematics* (1997, 1999) documents "have been developed to provide a rigorous and challenging curriculum for students" which includes a "broader range of knowledge and skills" (OMET, 1997, p. 3), and which "integrates appropriate technologies into the learning and doing of mathematics, while recognizing the continuing importance of students mastering essential arithmetic and algebraic skills" (OMET, 1999, p. 3). The *Grades 7-9 Mathematics Targeted Implementation and Planning Supports* (TIPS) document has been developed to further assist teachers, coordinators, and administrators as they continue to implement these reforms. This section will specifically provide educators with a variety of teaching strategies, that are supported by contemporary research in mathematics education.

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Assessment and Evaluation: Principles of Quality Assessment

What is Quality Assessment?

The National Council of Teachers of Mathematics (NCTM) Assessment Standards for School Mathematics (NCTM, 1995) suggest that good assessment should: enhance mathematics learning; promote equity; be an open process; and be a coherent process.

Quality assessment should be continual and promote growth in mathematics over time. Quality instruction and assessment should reflect the actions or competencies of mathematics and value process as well as product.

Important aspects of quality assessment

The Ontario Association of Mathematics Educators (OAME) document *Linking Assessment and Instruction in the Middle Years* (Onslow & Sauer, 2001) suggests that in order to be effective, assessment practices need to:

- be ongoing and an integral part of the learning-teaching process, giving students regular opportunities to demonstrate their learning;
- emphasize communication between teacher and student, parent and teacher, student and student;
- encourage students to reflect on their own growth and learning;
- enable the teacher to describe student growth in the cognitive, physical, social, and emotional domains, and
- provide opportunities for students to connect new learning to previous knowledge. (p. 1)

Quality assessment includes a variety of tools and strategies that assess both the processes and products of mathematics learning and serves a variety of purposes: diagnostic, formative, and summative.

Considerations regarding quality assessment

Teachers are continually assessing their students, both informally (through observation and listening) and formally. Teachers need to be sure that structured assessments are appropriate to the instructional tasks and are developmentally appropriate for the students. Teachers also need strategies for making assessment manageable so that it appears to be a seamless component of instruction.

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Assessment and Evaluation: Linking Assessment and Instruction

What is linking assessment and instruction?

Assessment that is aligned with curriculum and instruction is the most purposeful type of assessment. In linking assessment and instruction, assessment should be well connected to what students do in classrooms every day.

Important aspects of linking assessment and instruction

Research has shown that linking assessment with instruction increases students' knowledge. A meta-analysis by Black and William (1998), in which they reviewed approximately 250 studies, indicates that when teachers link assessment with instruction on a daily basis, students' learning generally improves. Gathering assessment regularly, using a variety of methods, provides teachers with wide-ranging indications of students' performance, and limits the bias and distortion that can occur through the use of a limited number of assessment strategies at the end of instruction. Multiple strategies, e.g., observations, portfolios, journals, rubrics, tests, projects, self- and peer-assessments, indicate to students that the teacher appreciates their daily contributions, values their reasoning and attitude toward mathematics, and does not base evaluations solely on their ability to be successful on tests.

Stenmark (1991) suggests that:

> As the forms of mathematics teaching become more diverse – including open-ended investigations, cooperative group activity, and emphasis on thinking and communication – so too must the form of assessment change. (p. 3).

Moving from instructional activities to assessment activities should be seamless: one should look like the other. Linking assessment with instruction allows teachers to involve students as responsible partners in their own learning. Understanding one's strengths and limitations is a key factor for growth and becoming an independent learner. When students are part of the decision making concerning how their work should be evaluated, e.g., development of a rubric, selection of work to be placed in their portfolio, they are more likely to understand the characteristics that constitute sound mathematical thinking, as well as what is expected of them.

Evidence from a variety of sources, in a variety of contexts, assists the teacher in accurate diagnosis and provides the information necessary for helping all students advance their understanding. Formative assessment throughout each day guides teaching, informing us of those who understand and those needing further time and practice. Assessment should guide and improve learning of mathematics and a teacher's teaching of mathematics on an ongoing basis. An over reliance on any one method, in limited contextual settings, tends to provide an imprecise depiction of a student's true achievement and disposition towards mathematics.

Considerations regarding linking assessment and instruction

Teachers must understand which assessment methods are appropriate and compatible with the teaching and learning approaches being used. All students deserve opportunities to demonstrate their mathematical abilities and attitudes. Some factors that can mislead interpretations of a student's mathematical understanding are culture, developmental level of the student, and language. Methods used to collect assessment information should not be overly time-consuming.

If students have a variety of tools available for instruction, these tools should be available for assessment. If students engage in instructional tasks in a variety of forms and groupings then these should also be part of the forms and groupings for assessment.

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Assessment and Evaluation: Balanced Assessment

What is balanced assessment?

Balanced assessment is assessment that allows students multiple opportunities to show what they know and can do. Balanced assessment considers different learning styles, different groupings of students, different task types, and focuses on a variety of mathematical concepts and processes.

Important aspects of balanced assessment

A program with balanced assessment should include a balance of:

- Categories of the Achievement Chart or mathematical processes – Assessment should focus not only on knowledge and understanding but should also ask students to demonstrate their ability to problem-solve and their application, and communication of mathematics.
- Task types – Balanced assessment is more than tests and quizzes and should include other assessments such as presentations, investigations, performance tasks, projects, portfolios, journals.
- Groupings – Most assessments are administered to students individually. Some tasks such as a performance task can be worked on in small groups. Students submit individual solutions to the task to demonstrate their understanding of the work.
- Purposes – Not all assessment should be summative. Diagnostic and formative assessment helps teachers make instructional decisions and provides feedback to students during the learning process.

Considerations regarding balanced assessment

Judicious selection of assessment tasks, strategies, and tools should help to provide a balance of assessment and still maintain its manageability.

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Assessment and Evaluation: Assessment Accommodations

What are assessment accommodations?

Assessment accommodations provide suitable opportunities for specific students to demonstrate their mathematical understanding and help them to develop a productive disposition. Assessment accommodations recognize that individual students may need changes to regular classroom assessment practices to allow them to show what they know and can do. Such accommodations are not limited to but could include:

- extended time on assessment tasks;
- assessment done orally rather than in writing, e.g., description, report, presentation, video;
- reducing the number of tasks used to assess the same concept or expectation;
- observation of achievement during instruction or seatwork as the focus rather than paper-and-pencil task;
- support during assessment tasks such as prompts to improve task completion and to activate prior knowledge;
- assessment tasks performed outside the normal classroom time under teacher supervision;
- progress submissions or observations as part of assessment to encourage success for students who are not submitting written work.

Important aspects of assessment accommodations

Teachers need to know the characteristics of the learners in the class to provide opportunities that match students' needs.

For specific students, assessment accommodations could be used for a variety of reasons:

- A student may be struggling due to some temporary circumstances, such as a broken arm, or math anxiety.
- A student's stage of processing a particular mathematical concept may be at a concrete level as opposed to the abstract level reached by many classmates.
- Students may require regular accommodations as part of their IEPs.
- ESL students may require accommodations in recognition of their stages of language acquisition.
- Some adolescent students are reluctant to hand in completed assignments and the teacher may need to gather and include observation data based on drafts done in class.
- A particular summative task may prove to be too challenging at the time it is given, resulting in the teacher deciding to provide formative feedback, further instruction and practice, then another summative assessment opportunity.

Considerations regarding assessment accommodations

Teachers need to be sure that assessment accommodations for individual students are integrated into the entire assessment process. Assessment planning should include ways to assimilate students without IEPs into the regular types of assessment strategies over time through explicit teaching of strategies and skills. In all cases, assessment accommodations are not a "watering down" of the subject but an opportunity for students to show what they know and can do in a way other than what was originally planned.

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Assessment and Evaluation: Evaluation

What is evaluation?

"Evaluation involves the judging and interpreting of the assessment data and, if required, the assigning of a grade." (Dawson & Suurtamm, 2003, p. 38) Evaluation should occur after sufficient time has been given to learn the relevant concepts and skills.

Important aspects of evaluation

"Evaluations of students' achievement at particular times have several characteristics. They are summative in nature, are usually designed to communicate to audiences beyond the classroom, and are often used to make important educational decisions for the students." (NCTM, 1995, p. 56).

To evaluate students' achievement, teachers summarize evidence from multiple sources to form a description or judgment of students' mathematical understanding as seen through the level of achievement of curriculum expectations. Evaluation requires the use of a teacher's professional judgment as well as assessment data.

Considerations regarding evaluation

There are several ways that teachers track and use student assessment data. In most cases, evaluation policies at the school board level help schools and teachers to maintain consistency. Consistency is further enhanced through teacher discussion and sharing. As well, professional development in assessment and evaluation helps teachers develop confidence in evaluation so that they can support their evaluation in dialogues with students, parents, and administrators.

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Adolescent Mathematics Learners

What are the characteristics of adolescent mathematics learners?

According to Reys et al (2003), students vary greatly in their development and readiness for learning and "teachers play a critical role in judging the developmental stage" of each student and in establishing "rich environments for students to explore mathematics at an appropriate developmental level." (p. 25) The adolescent mathematics learner (Grades 7-9) is experiencing great changes and challenges in several domains simultaneously. Intellectually, adolescents are refining their ability to form abstract thought, think symbolically, render objective judgments, hypothesize, and combine multiple reactions to a problem to achieve resolution. Wolfe (cited in Franklin, 2003), noted:

> By adolescence, students lose about 3 percent of the gray matter in their frontal lobe – this is a natural process where the brain 'prunes' away excess materials to make itself more refined and more efficient. Such changes could indicate why adolescents sometimes have difficulty prioritizing tasks or multitasking… . Some neuroscientists feel that the cerebellum may be responsible for coordination or cognitive activity in addition to muscle and balance coordination. If that's true, then it's possible that physical activity could increase the effectiveness of the brain and learning. (p. 4)

Physically, adolescents tend to mature at varying rates, e.g., girls developing physically earlier than boys; adolescents are often concerned about their physical appearance; and may experience fluctuations in metabolism causing extreme restlessness and/or lethargy. Emotionally, many adolescents are sensitive to criticism; exhibit erratic emotions and behaviour; feel self-conscious; often lack self-esteem; search for adult identity and acceptance; and strive for a sense of individual uniqueness. Socially, adolescents may be eager to challenge authority figures and test limits; can be confused and frightened by new school settings that are large and impersonal; are fiercely loyal to peer group values; and are sometimes cruel and insensitive to those outside the peer group.

Capitalizing on adolescent characteristics

Erlauer (2003) has suggested what she refers to as the *20-2-20 Rule for Reflection and Application at the Middle/High School Levels*. Twenty (20) minutes into the lesson, when students' attention is waning, the teacher has the students re-explain what they have just learned (e.g., brief class discussion, sharing with partner, or entry in student journals) with some form of feedback (e.g., from teacher or from classmates) to check for understanding. Within two (2) days of initial learning, the teacher requires students to review and apply the new information, e.g., mind-map, piece of writing, developing a related problem for classmate to solve. And within twenty (20) days, usually at the end of a unit, the teacher has the students reflect on what they have learned and apply the concepts/skills they have learned to a more involved project which is then shared with the whole class or a small group of students. (pp. 84-85)

Considerations regarding adolescent characteristics

Teachers of Grades 7-9 adolescent mathematics learners, should consider the following questions:

- Do students have a role in determining classroom rules and procedures?
- Do students feel safe to take risks and participate during mathematics learning?
- Do students have opportunities to move around and engage in situations kinesthetically?
- Are a variety of groupings used, for particular purposes?
- Do students have opportunities to discuss and investigate different ways of thinking about and doing mathematics?
- Do tasks have multiple entry points to accommodate a range of thinkers in the concrete-abstract continuum?

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Classroom Management

What is classroom management?

Classroom management can best be viewed as both an active and proactive phenomenon. Teachers have to make decisions on a daily basis concerning their teaching, learning and assessment processes, as well as the mathematical content they will stress. These and other important teacher choices directly determine student behaviour.

Important aspects of classroom management

Successful teachers establish a community of learners within a positive learning environment by: gaining the respect and trust of students, establishing clear and consistent routines, developing meaningful relationships with each student, understanding and accommodating the variety of student needs, and by selecting a full range of materials and strategies to teach mathematics.

Physical settings often convey to students whether their interactions are welcomed or if the teacher is the authority in transmitting the knowledge students are expected to learn. It is the teacher's attitude towards learning, more than the physical setting, which is likely to foster a community of learners. Teachers who want to empower students encourage them to take risks, and appreciate the value of their students *not knowing*, using these occasions as opportunities for growth rather than anxiety. Students are expected to make conjectures, justify their thinking, and respond to alternative perspectives respectfully, rather than memorize arbitrary rules that might not make sense to them.

Teachers present worthwhile tasks that challenge students and hold their interest. These tasks can often be solved in more than one way, and contain flexible entry and exit points, making them suitable for a wide range of students. Teachers who know their students' abilities and attitudes towards mathematics are likely to challenge them and pique their interest. They know what questions to ask and the appropriate time to ask them; when to remain silent, and when to encourage students to meet the challenge without becoming frustrated.

Considerations regarding classroom management

Kaplan, Gheen and Midgley (2002) conducted research with Grade 9 students and found that the emphasis on mastery and performance goals in the classroom is significantly related to students' patterns of learning and behaviour. Their conclusions are as follows:

> This study joins many others (see Urdan, 1997) that have pointed to the benefits of constructing learning environments in which school is thought of as a place where learning, understanding, improvement, and personal and social development are valued and in which social comparison of students' ability is deemphasized. (p. 206)

A balanced and active approach to teaching and learning allows students to become engaged in mathematics and to learn co-operative and self-management skills (Ares & Gorrell, 2002; Boyer, 2002; Brand, Dunn & Greb, 2002). Each lesson in TIPS includes a variety of student groupings and instructional strategies, depending on the learning task. Lesson headers (Minds On, Action!, Consolidate/Debrief) are reminders to plan active lessons that appeal to auditory, kinesthetic and visual learners, thereby, having a positive impact on classroom management.

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Communication

What is communication in a mathematics classroom?

Communication is the process of expressing ideas and mathematical understanding using numbers, pictures, and words, within a variety of audiences including the teacher, a peer, a group, or the class.

Important aspects of communication

The NCTM *Principles and Standards* (2000) highlights the importance of communication as an "essential part of mathematics and mathematical education" (p. 60). It is through communication that "ideas become objects of reflection, refinement, discussion and amendment" and it is this process that "helps build meaning and permanence for ideas and makes them public" (p. 60). The *Ontario Curriculum* (OMET, 1999) also emphasizes the significance of communication in mathematics, describing it as a priority of both the elementary school and the secondary school programs. It further states, "This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group" (p. 4).

Craven (2000) advocates for a strong emphasis on mathematical communication, providing ideas for the recording and sharing of students' learning (e.g., journal entry/learning log, report, poster, letter, story, three-dimensional model, sketch/drawing with explanation, oral presentation). He concluded by stating:

> Children must feel free to explore, talk, create, and write about mathematics in a classroom environment that honours the beauty and importance of the subject. Children must be empowered to take risks and encouraged to explain their thinking. Teachers must construct tasks that will generate discussion and provide an opportunity for students to explain their understanding of mathematical concepts through pictures, words, and numbers. (p. 27)

Whitin and Whitin (2002) conducted research by examining a fourth-grade class under the instruction of a progressive classroom teacher. The authors described how *talking* helped students explore, express their observations, describe patterns, work through difficult concepts, and propose theories; and how *drawing* and *writing* assisted students in recording and clarifying their own thinking. They concluded by noting that communication was "…enhanced, and the children's understanding developed, when mathematical ideas were represented in different ways, such as through a story, with manipulatives and charts, and through personal metaphors…. In these ways, children can become proficient and articulate in communicating mathematical ideas." (p. 211)

Considerations regarding communication

Franks and Jarvis (2001) maintained that new forms of communication, although potentially liberating and motivating, can initially be difficult and uncomfortable for teachers and students to explore. However, they noted that both of these groups, given sustained support, had rewarding experiences when asked to become playful yet thoughtful risk-takers. (p. 66)

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Concrete Materials

What are concrete materials?

Concrete materials (manipulatives) are physical, three-dimensional objects that can be manipulated by students to increase the likelihood of their understanding mathematical concepts.

Strengths underlying the use of concrete materials

Concrete materials have been widely used in mathematics education, particularly in the K-6 elementary panel, but also in the middle school years (see Toliver video series). The results of TIMSS (1996) showed that Grade 8 students from Ontario did less well in the areas of algebra and measurement than other areas. Do Grade 8 students make connections between abstract algebraic concepts and their more *concrete*, previously learned, arithmetic experiences? Chappell and Strutchens (2001) noted that:

> Too many adolescents encounter serious challenges as they delve into fundamental ideas that make up this essential mathematical subject [algebra]. Instead of viewing algebra as a natural extension of their arithmetic experiences, significant numbers of adolescents do not connect algebraic concepts with previously learned ideas. (p. 21)

In response to this situation, they recommended the use of concrete models, such as algebra tiles, to assist students in making connections and to facilitate mathematical understanding. Ross and Kurtz (1993) provided the following recommendations for teachers planning to implement a lesson involving manipulatives: (i) choose manipulatives that support the lesson's objectives; (ii) ensure significant plans have been made to orient students to the manipulatives and corresponding classroom procedures; (iii) facilitate the active participation of each student; and (iv) include procedures for evaluation that reflect an emphasis on the development of reasoning and processing skills, (e.g., listening to students talking about mathematics, reading their writings about mathematics, and observing them at work on mathematics. (pp. 256-257)

Considerations regarding concrete materials

Thompson (1994) maintained that professional development should model selective and reflective use of concrete materials, helping teachers to focus on "what students will come to understand" as a result of using manipulatives, rather than just on "what students will learn to do" (p. 557). Similarly, Moyer (2001) noted that manipulatives can be used in a rote manner with little or no learning of the mathematics, and that the effective use of concrete materials relies heavily on a teacher's background knowledge and understanding of mathematical representations. Other researchers have warned that manipulatives should not be over-used (Ambrose, 2002); should not be treated as a fun reward activity or trivialized through teachers' comments (Moyer, 2001); but, should be used effectively and selectively as a means of facilitating the ongoing transitions from the concrete (physical and visual) information involved in student learning to the more abstract knowledge of mental relationships and deep understandings. (Kamii, Lewis, & Kirkland, 2001)

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Differentiated Instruction

What is differentiated instruction?

Differentiated instruction is based on the idea that because students differ significantly in their interests, learning styles, abilities, and experiences, teaching strategies and pace should vary accordingly.

Strengths underlying differentiated instruction

The September 2000 issue of *Educational Leadership* focused almost exclusively on issues pertaining to differentiated instruction. Pettig (2000) noted that "differentiated instruction represents a proactive approach to improving classroom learning for all students," and that it "requires from us [teachers] a persistent honing of our teaching skills plus the courage to significantly change our classroom practices." (pp. 14, 18) Heuser (2000) presented math and science workshops, both teacher- and student-directed, as a viable strategy for facilitating differentiated instruction. He stated that the philosophy behind these workshops is founded on research and theory that support diverse learners' understanding of math and science, and can be summarized as follows: (i) children learn best when they are actively involved in math and science and physically interact with their environment; (ii) children develop a deeper understanding of math and science when they are encouraged to construct their own knowledge; (iii) children benefit from choice, both as a motivator and as a mechanism to ensure that students are working at an optimal level of understanding and development, (iv) children need time and encouragement to reflect on and communicate their understanding, and (v) children need considerable and varying amounts of time and experiences to construct scientific and mathematical knowledge. (p. 35) Notwithstanding the fact that "teachers feel torn between an external impetus to cover the standards [curriculum] and a desire to address the diverse academic needs," Tomlinson (2000) maintained:

> There is no contradiction between effective standards-based instruction and differentiation. Curriculum tells us what to teach: Differentiation tells us how. Thus, if we elect to teach a standards-based curriculum, differentiation simply suggests ways in which we can make that curriculum work best for varied learners. In other words, differentiation can show us how to teach the same standard to a range of learners by employing a variety of teaching and learning modes. (pp. 8, 9)

Considerations regarding differentiated instruction

Within differentiated instruction, the successful inclusion of all types of learners is facilitated. (Winebrenner, 2000, p. 2000) Karp and Voltz (2000) summarized differentiated instruction in the following way:

> As teachers learn and practice various teaching strategies, they expand the possibilities for weaving rich, authentic lessons that are responsive to all students' needs… the adherence to a single approach will create an instructional situation that will leave some students unravelled and on the fringe. (p. 212)

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Flexible Grouping

What is flexible grouping?

Flexible grouping refers to the practice of varying grouping strategies based on short-term learning goals that are shared with the students, then regrouping once goals are met. Groupings often include individual, partners, student- and teacher-led small groups, and whole-class configurations.

Strengths underlying flexible grouping

Linchevski and Kutscher (1998) suggest that during whole class discussions, the teacher could: develop conceptions about what mathematics is; create an appropriate learning environment and foster essential norms of classroom behaviour; legitimize errors as part of the learning process; and allow expressions of ambiguity. "These discussions also allow weaker students to participate, albeit many times passively via "legitimate peripheral participation" (Lave & Wenger, 1991) and "cognitive apprenticeship" (Brown, Collins, & Duguid, 1989), in a challenging intellectual atmosphere."

The Early Math Strategy (2003) suggests that a reason for independent mathematics, as well as shared and guided mathematics, is that "children demonstrate their understanding, practise a skill, or consolidate learning in a developmentally appropriate manner through independent work…Students need time to consolidate ideas for and by themselves." (p. 37)

Students need opportunities to learn from each other in small groups and pairs, to try ideas, practise new vocabulary, and construct their own mathematical understanding with others. Large homogeneous groups can engage in differentiated tasks including enrichment topics, remediation, and filling gaps for groups of students who were absent when a concept was taught.

Considerations regarding flexible grouping

In reporting the results of three studies that focused on the effects of teaching mathematics in a mixed-ability setting on students' achievements and teachers' attitudes, researchers Linchevski and Kutscher (1998) concluded that "it is possible for students of all ability levels to learn mathematics effectively in a heterogeneous class, to the satisfaction of the teacher." (p. 59) To examine high-achieving students' interactions and performances on complex mathematics tasks as a function of homogeneous versus heterogeneous pairings, Fuchs, Fuchs, et al. (1998) videotaped ten high achievers working with a high-achieving and a low-achieving classmate on performance assessments. Based on their findings, they recommended that "high achievers, when working on complex material, should have ample opportunity to work with fellow high achievers so that collaborative thinking as well as cognitive conflict and resolution can occur." (p. 251) However, they also noted that heterogeneous groupings can prove valuable when "used appropriately with less complex tasks, by providing maximal opportunities for high achievers to construct and low achievers to profit from well-reasoned explanations." (p. 251) Flexible grouping has much to offer, yet also demands careful planning.

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Graphic Organizers

What are graphic organizers?

Graphic organizers are visual representations, models, or illustrations that depict relationships among the key concepts involved in a lesson, unit, or learning task.

Strengths underlying the use of graphic organizers

According to Braselton and Decker (1994), mathematics is the most difficult content area material to read because "there are more concepts per word, per sentence, and per paragraph than in any other subject" and because "the mixture of words, numerals, letters, symbols, and graphics require the reader to shift from one type of vocabulary to another." (p. 276) They concluded by stating, "One strategy that is effective in improving content area reading comprehension is the use of graphic organizers. (Clarke, 1991; Flood, Lapp, & Farnan, 1986; Piccolo, 1987)" (p. 276) Similarly, DiCecco and Gleason (2002) noted that many students with learning disabilities struggle to learn in content area classes, particularly when reading expository text. Since, in their opinion, content textbooks often do not make important connections/relationships adequately explicit for these students, the authors recommended the use of graphic organizers to fill this perceived gap.

> Graphic organizers are one method that might achieve what textbooks fail to do. …They include labels that link concepts in order to highlight relationships (Novak & Gowin, 1984). Once these relationships are understood by a learner, that understanding can be referred to as relational knowledge. …Logically, if the source of relational knowledge is structured and organized, it will be more accessible to the learner (Ausubel, 1968). (p. 306)

Monroe and Orme (2002) presented two general methods for teaching mathematical vocabulary: meaningful context and direct teaching. Described as a powerful example of the latter method, the use of graphic organizers was further highlighted as being "closely aligned with current theory about how the brain organizes information" and as one of the more promising approaches regarding the recall of background knowledge of mathematical concepts. (p. 141)

Considerations regarding the use of graphic organizers

Monroe and Orme note that the effectiveness of a graphic organizer is limited by its dependence on background knowledge of a concept (Dunston, 1992). As an example, they explain that "if students have not encountered the concept of rhombus, a graphic organizer for the word will not help them to develop meaning." (p. 141) Also noteworthy on this topic is the fact that many software programs and websites are now being developed which provide students with digital versions of graphic organizers and *virtual manipulatives*, e.g., interactive animations of 3-dimensional, mathematical learning objects, an increasing number of which are becoming available free of charge for educators and parents.

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TIPS References

Grade 7: Venn diagrams, Days 12 & 13
Grade 8: Ranking Ladder, Day 3; Yes/No examples, Day 11; Tree Diagram, Day 18
Grade 9 Applied: Placemat, Unit 2, Day 3

Japanese Lesson Structure

What are the typical elements of a Japanese mathematics lesson?

A typical Japanese mathematics lesson features the following components: (i) the posing of a complex, thought-provoking problem to the class; (ii) individual and/or small group generation of possible approaches for solving the problem; (iii) the communication of strategies and methods by various students to the class; (iv) classroom discussion and collaborative development of the mathematical concepts/understandings; (v) summary and clarification of the findings by the teacher; (vi) consolidation of understanding through the practice of similar and/or more complex problems.

Strengths underlying the Japanese lesson structure

Since the release of the Third International Math and Science Study (TIMSS) results in 1996, nations have been devoting a great deal of time, resources, and research to the process of unravelling the diverse findings, and exploring various methods for national mathematical and scientific reform. Since Japanese mathematics students outperformed their global peers, attention has focused on the methods of instruction and classroom management that have been adopted by Japanese educators. Lessons in Japanese classrooms were found to be remarkably different from those in Germany and the U.S., promoting students' understanding, while U.S. and German teachers seemed to focus more exclusively on the development of skills. (Martinez, 2001; Roulet, 2000; Stigler & Hiebert, 1997)

The considerable time spent in Japanese classrooms on inventing new solutions, engaging in conceptual thinking about mathematics, and communicating ideas has apparently paid rich dividends in terms of students' understanding and achievement. It is somewhat ironic to note that in light of the fact that TIMSS has been criticized as being, overly skill-based as opposed to featuring more problem-solving content, Japanese students, who have experienced the types of reforms promoted by groups such as the National Council of Teachers of Mathematics, also outperformed the world on more *traditional* mathematics questions. It appears that the deeper understandings cultivated through these methodologies are not at the expense of technical prowess; rather this form of instruction seems to strengthen both procedural and conceptual knowledge.

Considerations regarding Japanese lesson planning

Japanese teachers regularly take part in lesson study and inquiry groups, producing gradual but continual improvement in teaching. (Stigler & Hiebert, 1999) Watanabe (2002) noted that "a wide range of activities characterizes this kind of professional development, offering teachers opportunities to examine all aspects of their teaching – curriculum, lesson plans, instructional materials, and content." (p. 36) He further recommended that in order to learn from Japanese lesson study, North American teachers should: (i) develop a culture through regular and collective participation; (ii) develop the habit of writing an instruction plan for others; (iii) develop a unit perspective; (iv) anticipate students' thinking; and (v) learn to observe lessons well.

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Mental Mathematics and Alternative Algorithms

What is meant by mental mathematics and alternative algorithms?

People who are mathematically literate can generally compose (put together) and decompose (take apart) numbers in a variety of ways. Having flexible thinking skills empowers students and frees them to invent strategies that make sense to them.

Example:

Students who use the traditional subtraction algorithm for 6000 – 1, often get the answer wrong, and generally don't have a good sense of number or a positive attitude towards mathematics.

When asked to multiply 24 by 25 many people who are comfortable playing with numbers use strategies such as divide 24 by 4 (recognizing 4 quarters make one dollar and so 24 quarters equals $6 or 600 cents); or add 240+240+120 (decomposing: 24 x 10 + 24 x 10 + 24 x 5); or 625 – 25 (they know 25 x 25 then subtract 25).

Strengths underlying mental mathematics and alternative algorithms

Research projects have shown that young people who invent their own alternative algorithms to solve computation questions generally have a firm sense of number and place value (Kamii & Dominick, 1997; Van de Walle, 2001). When students invent their own strategies for solving computation questions, the strategies tend to be developed from knowledge and understanding rather than the rote memorization of the teacher's method. As Kamii and Dominick (1997) explain, knowledge is developed from within and young people can then trust their own powers of reasoning. Kamii and others have shown that students who invent their own algorithms tend to do as well as other students on standardized computation tests, and also have a far better understanding of what is happening mathematically.

Many computational errors and misconceptions are based on students' misunderstandings of traditional algorithms. When subtracting 48 from 72 they often write 36, subtracting the smaller digits from the larger ones. When asked why zeros are included when using the traditional algorithm to multiply 25 by 33, few students can explain that they multiplying 25 by 3 and 25 by 30. They have an even more difficult time explaining why these two sums are added together. Misconceptions are often the result of following procedures without understanding. Alternative algorithms encourage students to make sense of what they are doing rather than accepting rules based on faith.

Considerations regarding alternative algorithms

Many mathematics educators believe that students should be taught traditional algorithms after or along with the opportunity to invent their own algorithms. Traditional algorithms are an essential part of mathematics learning and should be taught, but only after students have developed understanding of the concept and shared their own approaches to the problem.

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Metacognition

What is metacognition?

Metacognition is the awareness and understanding of one's own thought processes; in terms of mathematics education, the ability to apply, evaluate, justify, and modify one's thinking strategies.

Important aspects of metacognition

Metacognition and its implications for mathematics education are described in the National Council of Teachers of Mathematics *Principles and Standards* (2000) document:

> Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems (Bransford et al. 1999). Such reflective skills (called *metacognition*) are much more likely to develop in a classroom environment that supports them. Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as "Before we go on, are we sure we understand this?" "What are our options?" "Do we have a plan?" "Are we making progress or should we reconsider what we are doing?" "Why do we think this is true?" Such questions help students get in the habit of checking their understanding as they go along. (p. 54)

Kramarski, Mevarech and Lieberman (2001) noted that "For more than a decade, metacognition researchers have sought instructional methods that use metacognitive processes to enhance mathematical reasoning." (p. 292) For example, Pugalee (2001) described how metacognition has been shown to complement problem solving, enhancing decisions and strategies such as predicting, planning, revising, selecting, checking, guessing, and classifying. (p. 237)

Considerations regarding metacognition

Although many students are able to discover thinking and problem-solving strategies independently or detect unannounced strategies that they see others use, some students who have "metacognitive deficits may not even be aware that others are using strategies to successfully complete the task at hand." (Allsopp et al., 2003, p. 310) Moreover, these deficits "become more evident as students are expected to apply strategies they have learned to new situations, concepts, or skills." (p. 310) In light of this reality, and the fact that young people typically like to talk about their thinking in the classroom, researchers Stright and Supplee (2002) recommend that small-group instructional contexts be used often to encourage *math talk* and to facilitate the sharing of varied models of thinking among students. (p. 237) Part of consolidation and debriefing should provide opportunities for students to reflect on the lesson, through writing in journals and/or talking in small groups, or whole-class discussions.

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Providing Feedback

What is meant by providing feedback?

Providing feedback involves the comprehensive and consistent communication of information regarding a student's progress in mathematics learning, to both the student and the parent.

Important aspects of providing feedback

Mathematics educators are responsible for not only preparing and presenting quality instruction, but also for developing and implementing effective methods of assessment and reporting. Providing regular formative and summative feedback to both students and parents is of importance to help students adjust to the demands of the mathematics program. There is perhaps nothing worse than arriving at the end of a term or semester only to find that one's marks, or one's child's marks, in mathematics are dismally below average, yet without any prior feedback being received.

One difficult area for teachers is to know how much assistance to offer students on a particular mathematics problem or task, without giving away too much information. Chatterley and Peck (1995), in their article entitled, "We're Crippling our Kids with Kindness!", noted the following:

> The key is experience. We cannot, even though with the most kindly of intentions, exclude students from those experiences that come from struggling with a problem. …We cannot cripple our students mentally by taking away from them the struggles that must come before understanding is brought to fruition. If we understand the process necessary to provide the referents within the minds of our students, we will cease to mentally cripple them by being overly kind and sympathetic and by helping too much and often far too soon. (pp. 435-436)

Kewley (1998) emphasized the importance of this notion of *cognitive dissonance*, or psychological discomfort, by referring to the work of several key educational theorists:

> Both Piaget (Ginsburg & Opper, 1979) and Vygotsky (Wertsch & Stone, 1985) believed that disequilibrium was a process necessary to learning, because if everything goes according to plan, nothing rises to the level of consciousness. Disequilibrium, or cognitive imbalance, is a state that occurs when the learner is unable to assimilate an experience or achieve a goal. It motivates the student's search for better knowledge and a valid solution. (pp. 30-31)

Assistance during the lesson or task must be meaningful and encouraging, but not overstated; teachers must carefully provide students with enough feedback to help them move forward by themselves (see Scaffolding).

Considerations regarding providing feedback

Both the NCTM *Principles and Standards* (2000) and *The Ontario Curriculum* (1997, 1999) highlight the importance of parental involvement in mathematics education. Because teachers, students, and parents/guardians are considered partners in schooling, the provision of consistent feedback regarding student progress becomes a vital link connecting all members of this team. As Onslow (1992) noted:

> If we want parents to understand mathematics as more than the procedural drills of arithmetic then they must be provided with opportunities to explore their children's mathematics programme. In this way, parents will be in a position to understand not only what we are teaching but why we are teaching the way we do. (p. 25)

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Questioning

What is questioning?

Teacher questioning and teacher listening are closely linked skills that are employed daily in the mathematics classroom to guide both teaching and learning, facilitate participation, and to stimulate higher-order thought.

Important aspects of questioning

Nicol (1999) conducted research regarding questioning with pre-service teachers. She found that prospective teachers seemed to "struggle with not only how they might ask students questions but also what they might ask and for what purpose." (p. 53) She concluded that "As prospective teachers… began to consider students' thinking and create spaces for inquiry through the kinds of questions posed, they began to see and hear possibilities for mathematical exploration that evolved as their relationship with mathematics and with students changed." (p. 62) Defining a higher order question as a query that asks students to respond at a higher level than factual knowledge, Wimer et al. (2001) conducted research surrounding the *higher order* questioning of boys and girls in elementary mathematics classrooms. They noted in summary that "higher level and lower level questions have their place in the classroom; vigilant teachers employ both types when the need arises." (p. 91)

After observing the low levels of achievement of many of his middle school students, Reinhart (2000) decided to implement changes in his teaching methods. He noted, "It was not enough to teach better mathematics; I also had to teach mathematics better. Making changes in instruction proved difficult because I had to learn to teach in ways that I had never observed or experienced, challenging many of the old teaching paradigms." (p. 478) Understanding his students proved helpful in this process:

> Getting middle school students to explain their thinking and become actively involved in classroom discussions can be a challenge. By nature, these students are self-conscious and insecure. This insecurity and the effects of negative peer pressure tend to discourage involvement. To get beyond these and other roadblocks, I have learned to ask the best possible questions and to apply strategies that require all students to participate. (pp. 478-479)

Reinhart offered five suggestions for implementing these positive, yet difficult changes: (i) never say anything a kid can say, (ii) ask good questions, i.e., that require more than recalling a fact or reproducing a skill; the best questions are open-ended, (iii) use more process questions (i.e., that require the student to reflect, analyze, and explain his/her thinking and reasoning) than product questions (i.e., that require short answers, yes/no responses, or rely almost completely on memory), (iv) replace lectures with sets of questions, and (v) be patient, i.e., wait time is very important; increasing it to five seconds or longer results in better responses. (p. 480)

Considerations regarding questioning

Although the Socratic method of questioning and answering has existed as a longstanding and effective mathematics teaching strategy, it is best used as one method among several. Open-ended questioning, some of it creating disequilibrium, and informal classroom discussions regarding mathematical thinking and processes are valuable.

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TIPS References

TIPS for Teachers: "Questioning" includes: Clarifying Questions, Prompting Questions, Reflective Questions, Strategic Questions, and Inquiry/ Big Questions.

Scaffolding

What is scaffolding?

Scaffolding is a metaphor that is used as a framework to describe how teachers can guide students through a learning task. Scaffolds may be tools, such as cue cards, analogies, and models; or techniques, such as teacher modeling, prompting, or thinking aloud.

Strengths underlying scaffolding

Research has shown that scaffolding is most useful in situations where students are at a loss as to what they should do, but can proceed, and accomplish a task that is just outside their level of competency, with assistance. The successful use of scaffolding requires the educator to determine the background knowledge of each student, and to develop a comfortable, working rapport with each student. One research study in which a mathematics teacher used a "thinking aloud" strategy as a form of scaffolding is described in the following vignette:

> In a mathematics study by Schoenfeld (1985), the teacher thought aloud as he went through the steps in solving procedures he was using (for example, making diagrams, breaking the problem into parts). Thus, as Schoenfeld points out, thinking aloud may also provide labels that students can use to call up the same processes in their own thinking. …Through modeling and thinking aloud, he applied problem-solving procedures and revealed his reasoning about the problems he encountered. Students saw the flexibility of the strategies as they were applied to a range of problems and observed that the use of a strategy did not guarantee success. …As individual students accepted more responsibility in the completion of a task, they often modeled and thought aloud for their less capable classmates. Not only did student modeling and think-alouds involve the students actively in the process, but it allowed the teacher to better assess student progress in the use of the strategy. Thinking aloud by the teacher and more capable students provided novice learners with a way to observe "expert thinking" usually hidden from the student. (Rosenshine & Meister, 1992, p. 28)

Although scaffolding was developed as an educational theory particularly in response to the specific needs of students with learning disabilities (Stone, 1998), it has more recently seen application in both general and adult education, as an affective strategy for all learners (Graves et al., 1996). Furthermore, this strategy has also been found effective in teaching higher-order cognitive skills. Rosenshine and Meister (1992) described six components that they believe comprise the successful teaching of higher-order cognitive skills via scaffolding: (i) presenting a new cognitive strategy, (ii) regulating difficulty during guided practice, (iii) varying the context for practice, (iv) providing feedback, (v) increasing student responsibility, and (vi) providing student responsibility. (pp. 26-32)

Considerations regarding scaffolding

Stone (1998) documented some cautionary notes. Teachers should be aware that scaffolding is meant to be a temporary, as opposed to long-term strategy; and that scaffolding requires adaptation for individual learners. (pp. 349-350) Scaffolding should not focus exclusively on teacher-directed methods that provide more information than is necessary.

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Student-Centred Investigations

What are student-centred investigations?

Student-centred investigations, referred to as rich investigations or rich tasks, are learning contexts which require students to explore mathematics through inquiry, discovery, and research.

Strengths underlying student-centred investigations

Chapin (1998) defined a mathematical investigation as a "multidimensional exploration of a meaningful topic, the goal of which is to discover new ways of thinking about the mathematics inherent in the situation rather than to discover particular answers." (p. 333) She elaborated on their significance:

> Investigations afford students an opportunity for sustained, in-depth study of a topic; students must make sense of their observations and synthesize and analyze their conclusions. …In addition, connections among different areas of mathematics (e.g., algebra, geometry, statistics) can be made, since students tend to bump into related ideas and concepts while pursuing the investigative questions. The "bumping" phenomenon enables a teacher to pursue related topics in parallel with the mathematical investigation – presenting a context for connecting ideas, for clarifying concepts, or for teaching new material. Finally, mathematical investigations can assist in establishing a classroom environment that supports inquiry. Students are expected to explore questions and engage in relevant discourse. By working slowly through many levels of questions and responses, students begin to experience the importance of careful reasoning and disciplined understanding. (pp. 333-334)

Flewelling and Higginson (2000) have developed similar ideas in their work regarding rich tasks. Creativity is presented as a salient feature of these explorations, in contrast to many traditional tasks which "ask students to follow given recipes to expected end-points, giving students little opportunity to consider alternatives and be creative." (p. 18) Rich learning tasks are designed in such a way that "different students are able to demonstrate (very) different kinds and levels of performance." (p. 18)Researchers Ares and Gorrell (2002) interviewed teachers and students from five middle schools to gain insight into perceptions surrounding meaningful learning experiences. They reported that: "The overriding message from students is that active learning, rather than passive listening, reading, and note-taking, draws them into subjects and deepens their understanding and appreciation of what they are learning." (p. 270) Based on their study, they concluded that, "…the assumption underlying management techniques should be that students and teachers value the same things: productive interactions centred on substantive learning." (p. 275)

Considerations regarding student-centred investigations

Some educators advocate the use of student-centred investigations throughout the entire curriculum, introducing *traditional* mathematics skills, when appropriate, to support the rich task explorations. Others view investigations as one successful strategy among many, to be used several times throughout the term, semester, or course. Teachers new to the idea are encouraged to plan and try one investigation of interest, reflect on and discuss the task with colleagues, and then make changes to the investigation before using it again, based on this professional dialogue.

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Teacher-Directed Instruction

What is teacher-directed instruction?

Teacher-directed instruction involves teaching rules, concepts, principles, and problem-solving strategies in an explicit fashion, providing a wide range of examples with extensive review and practice.

Strengths underlying teacher-directed instruction

According to Baker, Gersten, and Lee (2002), who synthesized empirical research on teaching mathematics to low-achieving students, "research suggests that principles of direct or explicit instruction can be useful in teaching mathematical concepts and procedures." (p. 68) Monroe and Orme (2002), focusing on the development of mathematical vocabulary, stated the following:

> Direct teaching of selected vocabulary has been advocated for many years (Gray & Holmes, 1938, cited in Chall, 1987; Moore, Readence, & Rickelman, 1989) and is supported by Vacca and Vacca (1996) and Klein (1988). Vacca and Vacca (1996) assert that the most important vocabulary words "need to be taught directly and taught well." (p. 136) Klein expresses the idea that direct teaching of vocabulary will guide students to assign deeper meaning to words. (p. 140)

The authors also noted the existence of poor and ineffectual methods of teaching vocabulary via direct-instruction, such as the "definition-only" approach. Therefore, they advocated a balanced approach for teaching mathematical vocabulary "that combines meaningful context and direct teaching through the use of a graphic organizer." (p. 141) Wilson, Majsterek, and Simmons (1996) compared the effects of computer-assisted versus teacher-directed instruction on the multiplication performance of elementary students with learning disabilities. Although their findings suggested that for these students teacher-directed procedures were the more efficient and effective method of achieving basic fact mastery, they also recommended a balanced and context-sensitive approach, involving both teacher-directed and computer-assisted instruction. (p. 389) Whereas the results of Stright and Supplee's (2002) research suggested that students are more self-regulated learners in small group and seat work settings, the authors also concluded by stating, "In order for children to truly become self-regulated learners, the classroom should include all three contexts [teacher-directed, individual seat work, and group work] to provide direct instruction, independent practice, and the opportunity to practice metacognitive skills in a social context (Slavin, 1987)." (p. 242)

Considerations regarding teacher-directed instruction

Kewley (1998) pointed out that, although research indicates that teacher-directed instruction promotes learning, the method also has inherent problems. For example, the "superior adult mentality tends to dominate the proceedings, suppressing the reciprocity of ideas and their coordination. The children are then less stimulated to clarify their own ideas, an essential element in gaining understanding and eventually taking ownership of a concept." (p. 31) The author asserted that "by varying instructional methods there is a greater possibility that teachers will meet the needs of all students, since some may learn better through one set of mechanisms coming into play than another." (p. 31)

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Technology

What is technology?

Technology, a term derived from the Greek *teknologia* meaning "systematic treatment," encompasses both a wide range of products, e.g., calculators, software, hardware, and related systematic processes.

Strengths underlying the use of technology

Gilliland (2002) noted that although machines do not think, calculators can "take the drudgery out of computation by performing low-level tasks in mathematics." (p. 50) In referring to related research, she further explained:

> Students who learn paper-and-pencil techniques in conjunction with the use of four-function calculators or other technology, and are tested without calculators, perform as well as, or better than, those who do not use technology in class. Appropriate use of calculators does not result in the atrophy of computational skills; instead, it provides an impetus and opportunity for students to focus on conceptual learning (Heid, 1997). (p. 50)

Despite this type of research, only 18% of Grade 3 students and fewer than 10% of Grade 6 students use calculators on a regular basis in Ontario schools (EQAO, *Report of Provincial Results*, 2002, pp. 30, 39).

Being aware of the increased emphasis on mathematics problem solving and the poor performance of students with learning disabilities, Babbitt and Miller (1996) documented and explored appropriate methods for teaching these skills with greater effectiveness and efficiency. (p. 392) The researchers recommended the use of computers, and specifically hypermedia, i.e., digital environments that go beyond text and traditional computer-assisted instruction to incorporate sound, animation, photographic images, and video clips in sophisticated ways, to teach mathematical problem solving to students with learning disabilities. (pp. 393-395)

Considerations regarding the use of technology

The advent of increased technology in the mathematics classroom does not guarantee improved teaching or increased learning. Although complex in nature, and possessing expansive possibilities, technological products such as calculators and software packages are only as effective as the teacher using them. Among the many considerations regarding the implementation of technology that face administrators and teachers in the 21st Century, the two that are of perhaps greatest import are resources and training. Extended and innovative teacher training programs (Woolley, 1998) are most desirable. Dedicated teachers should be encouraged to continue exploring these strategies and tools, keeping in mind the reality that often the students themselves are a rich resource when it comes to technology in the classroom.

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van Hiele Model

What is the van Hiele model?

The van Hiele level approach is based on the 1950's work of a Dutch couple, Dina van Hiele-Geldof and Pierre van Hiele, who developed a model that described five distinctive geometry learning levels: visualization, analysis, informal deduction, formal deduction, and rigor. Being neither age-dependent nor content-based, the van Hiele levels describe how students think about geometry and how this thinking changes over time as students become increasingly competent.

Strengths underlying the use of the van Hiele model

Van de Walle (2001) described the five levels in the following way:

Visualization: Students recognize and name figures based on the global, visual characteristics of the figure and are able to make measurements and talk about simple properties of shapes.

Analysis: Students are able to consider all shapes within a class rather than a single shape, but are unable to understand the intricacies of categorical definitions.

Informal Deduction: As students begin to be able to think about properties of geometric objects without the constraints of a particular object, they are able to develop relationships between and among these properties.

Formal Deduction: Students are able to examine more than just the properties of a shape, recognizing the significance of an axiomatic system and able to construct geometric proofs.

Rigor: Students become increasingly capable of understanding a complex system complete with axioms, definitions, theorems, corollaries, and postulates. (pp. 309-310)

Because the van Hiele approach assumes that students progress through the five levels sequentially, and that lessons must be delivered at a level that matches this progress, the necessity for teachers to develop a clear picture of prior knowledge and ongoing student understanding is underscored. According to Malloy (1999): "By the time that students enter the middle grades, most of them are between the concrete [first and second] and informal deduction [third] levels defined by the van Hieles. (p. 87) However, teachers often talk about geometry using third or fourth level language that students cannot understand, leading to a mismatch in teaching and learning. An awareness of this disjuncture allows teachers to modify their presentation of geometric concepts in order to appropriately engage students.

Considerations regarding the use of the van Hiele model

To illustrate how educators can help students progress from one van Hiele level to another, Malloy (1999) provided the following recommendations:

> The van Hiele model suggests using five phases of instruction to help students in this progression. Students first gather information by working with examples (e.g., finding the perimeter of shapes), then they complete tasks that are related to the information, such as adding tiles to the figure to increase perimeter. The students become aware of relationships and are able to explain them. Finally, students are challenged to move to more complex tasks and to summarize and reflect on what they have learned. The language used by teachers and students is important for students' progression through the levels from concrete to visual to abstract (Fuys, Geddes, and Tischler, 1988. p. 89)

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